THE SEISMIC EFFECT OF IMPACTS ON ASTEROID SURFACE MORPHOLOGY

by

James Edward Richardson Jr.

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As members of the Final Examination Committee, we certify that we have read the
dissertation prepared by James Edward Richardson, Jr.
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and recommend that it be accepted as fulfilling the dissertation requirement for the
Degree of Doctor of Philosophy

H. Jay Melosh  5/13/05
Richard Greenberg  5/13/05
Elizabeth P. Turtle  19 May 2005
Clement G. Chase  5/13/05
Jon D. Pelletier

Final approval and acceptance of this dissertation is contingent upon the
candidate’s submission of the final copies of the dissertation to the Graduate College.
I hereby certify that I have read this dissertation prepared under my direction and
recommend that it be accepted as fulfilling the dissertation requirement.

Dissertation Director: H. Jay Melosh  Richard Greenberg

5/13/05
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SIGNED:
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ABSTRACT

Impact-induced seismic vibrations have long been suspected of being an important surface modification process on small satellites and asteroids. In this study, I use a series of linked seismic and geomorphic models to investigate the process in detail. I begin by developing a basic theory for the propagation of seismic energy in a highly fractured asteroid, and I use this theory to model the global vibrations experienced on the surface of an asteroid following an impact. These synthetic seismograms are then applied to a model of regolith resting on a slope, and the resulting downslope motion is computed for a full range of impactor sizes. Next, this computed downslope regolith flow is used in a morphological model of impact crater degradation and erasure, showing how topographic erosion accumulates as a function of time and the number of impacts. Finally, these results are applied in a stochastic cratering model for the surface of an Eros-like body (same volume and surface area as the asteroid), with craters formed by impacts and then erased by the effects of superposing craters, ejecta coverage, and seismic shakedown. This simulation shows good agreement with the observed 433 Eros cratering record at a Main Belt exposure age of 400 ± 200 Myr, including the observed paucity of small craters. The lowered equilibrium numbers (loss rate = production rate) for craters less than ~100 m in diameter is a direct result of seismic erasure, which requires less than a meter of mobilized regolith to reproduce the NEAR observations.

This study also points to an upper limit on asteroid size for experiencing global, surface-modifying, seismic effects from individual impacts of about 70-100 km (depending upon asteroid seismic properties). Larger asteroids will experience only local seismic effects from individual impacts.

In addition to the study of global seismic effects on asteroids, a chapter is also included which details the impact ejecta plume modeling I have done for the Deep
Impact mission to the comet Tempel I. This work will also have direct application to impacts on asteroids, and will be used in the future to refine the cratering history modeling performed thus far.
CHAPTER 1

INTRODUCTION

1.1 Background

In the spring of 1964, the Good Friday earthquake struck the Prince William Sound area of Alaska, with a magnitude of 8.4 on the Richter Scale (9.2 Moment Magnitude) and a duration of 240 seconds (Wood, 1966). One of the more interesting features of this earthquake was that more than half of the property damage was caused by slope failure, rather than structural failure or other earthquake effects (Keefer, 1984). This fact prompted a wave of interest (particularly among civil engineers) in the mechanics of slope failure under dynamic loading conditions, which, in turn, produced a string of papers on the topic. Two of the more notable publications on the subject involved an experimental shake-table investigation of the downslope movement of soil under seismic shaking conditions (Goodman and Seed, 1966), and numerical and analytical modeling of the same phenomena (Newmark, 1965). I shall return to both of these papers later in this work.

At about the same time, the Ranger images of the Earth’s moon revealed the downslope flow of material on lunar slopes in the form of slides, slumps, and creep processes, and impact-induced seismicity was proposed as a potential cause (Titley, 1966). Further analysis showed that while impacts large enough to produce complex craters and basins can produce widespread seismic effects on the Moon (Schultz and Gault, 1975a,b), the impacts that form small, simple craters will affect only local areas of high slope (Houston et al., 1973). The Mariner and Viking images of the martian moons, Phobos and Deimos, opened up this same question with regard to small satellites and asteroids. J. Arnold is given credit for first suggesting
that the downslope movement of regolith could be an important surface process, despite the tiny surface gravitational fields on these small bodies (Chapman, 1978). In the same year, Cintala et al. (1978) published two of the primary reasons why impact-induced seismic shaking of a small body is an attractive surface modification mechanism. First, the small volume of the body keeps the concentration of seismic energy high even after the seismic energy injected by an impact has completely dispersed throughout the target body. Second, the very low surface gravity of a small body \( g_a < 10^{-3} g_{\text{earth}} \) permits relatively small seismic accelerations to destabilize material resting on slopes, where destabilization begins at 0.2-0.5\( g_a \) for loose regolith (Lambe and Whitman, 1979).

Modeling of the elastic stresses and seismic effects of large impacts on small bodies began with the work of Fujiwara (1991) and Ivanov (1991), who investigated the formation of the Stickney impact basin on Phobos and the resulting stress features (and rudimentary seismic effects) on its surface. In a separate analysis, Asphaug and Melosh (1993) performed hydrocode modeling of the impact that produced the Stickney basin, and as a by-product estimated the resulting velocities imparted to a hypothetical regolith layer resting on the surface of the moon, along with approximate ballistic travel distances: an effect called seismic ‘jolt’ (Nolan et al., 1992). Asphaug and Melosh (1993) thus produced the first published computation of impact seismic effects on the regolith layer of a small body. Following up on this method, M. Nolan produced more extensive seismic jolt estimates for the surfaces of asteroid 951 Gaspra (Greenberg et al., 1994) and 243 Ida (Greenberg et al., 1996), which were imaged at close range by the Galileo spacecraft in 1991 and 1993, respectively. These analyses indicated that impacts producing kilometer-sized craters on these small asteroids can have severe effects on their cratering records, erasing most of the craters below a few hundred meters in diameter when a thick, loose regolith layer exists. Further three-dimensional modeling using an Ida shape model demonstrated how severe antipodal surface damage (such as groove formation) can
result from large impacts, even when the target body has a highly irregular shape (Asphaug et al., 1996).

The NEAR-Shoemaker orbiter mission to the asteroid 433 Eros (2000-2001) revealed a heavily cratered surface, covered with a veneer of loose regolith (tens of meters thick in places) and peppered with numerous boulders (Veverka et al., 2001; Robinson et al., 2002). This regolith layer displays direct evidence of downslope movement in several forms (see Figs. 1.1, 1.2, 1.3, and 1.4): slumps and debris aprons at the base of steep slopes, bright streaks of freshly exposed material on crater walls, the pooling of regolith in topographic lows, a large number of degraded craters, and a deficit of craters less than $\sim 100$ m in diameter as extrapolated from larger crater sizes (Veverka et al., 2001; Chapman et al., 2002; Cheng et al., 2002a; Thomas et al., 2002; Robinson et al., 2002). One potential explanation for these phenomena is seismic reverberation of the asteroid following impact events (Veverka et al., 2001), which is potentially capable of destabilizing slopes, causing regolith to migrate downslope, and degrading or erasing small craters. In this study, I use the high-resolution, global-coverage, Eros data set to provide important modeling constraints in investigating the phenomena of impact-induced seismic shaking, such that any adequate model of impact generated seismic effects must be able to explain and match this observational evidence. In addition to the more obvious indications of downslope regolith motion, I will pay particular attention to the Eros cratering record, especially the large number of degraded craters and the paucity of small craters (see Fig. 1.5).

1.2 Minimum impactor size for global seismic effects (I)

Why is seismic shaking an attractive mechanism for small asteroids? I can illustrate both points raised by Cintala et al. (1978) by equating the seismic energy injected by an impactor, which is a small fraction of the impactor's kinetic energy, with the seismic energy necessary to produce accelerations that exceed the asteroid's surface
Figure 1.1: An indication of downslope regolith motion on 433 Eros, imaged by the NEAR-Shoemaker spacecraft, in the form of bright streaks of freshly exposed material on a large crater wall as the darker material moves downslope (MET 154409710, 14.79 W, 14.21 S, 2.67 m/pixel).
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Figure 1.5: Two figures from Chapman et al. (2002), showing two relative size-frequency distributions (R-plots) of craters on the surface of 433 Eros (this plotting format is defined in Arvidson et al. (1979)). The upper plot was produced from the ‘nominal’ orbit data from the NEAR spacecraft, while the lower plot was produced from a number of low-altitude flybys (LAFs) conducted near the end of the mission. Note the steady drop-off in crater density as one moves to smaller sizes, beginning at a size of ~ 100 meters. One of the key questions of this study is to investigate if this paucity of small craters could be due to degradation and erasure from impact induced seismic effects.
gravity $g_a$ throughout its volume, and therefore destabilize all slopes on the surface. This calculation will give me an analytical estimate of the minimum diameter of impactor necessary to cause global seismic effects on the surface of a target asteroid.

The seismic energy injected by an impactor $E_i$ is given by:

$$E_i = \eta E_k = \frac{1}{12} \eta \pi \rho_p v_p^2 D_p^3,$$

(1.1)

where $\eta$ is an impact seismic efficiency factor (addressed in further detail below), $E_k$ is the kinetic energy of an impactor, $\rho_p$ is the mean projectile density, $v_p$ is the impact velocity, and $D_p$ is the spherical projectile diameter. Eq. 1.1 gives us one half of our desired expression.

The average seismic strain energy per unit volume of rock $\epsilon_d$ (energy density), expressed in terms of the maximum half-cycle amplitude $A$ resulting from a single (primary) seismic frequency is derived in Lay and Wallace (1995) as:

$$\epsilon_d = \pi^2 \rho_a \frac{A^2}{\tau^2},$$

(1.2)

where $\rho_a$ is the mean density of the asteroid and $\tau$ is the period of the seismic waves. Placing Eq. 1.2 in terms of the seismic frequency in Hertz $f$ rather than period $\tau$ gives:

$$\epsilon_d = \rho_a \pi^2 f^2 A^2.$$  

(1.3)

Since the maximum acceleration $a$ experienced by the medium can be expressed as a function of the maximum displacement $A$ and frequency $f$ ($a = -4\pi^2 f^2 A^2$), I can rewrite Eq. 1.3 in these terms:

$$\epsilon_d = \frac{\rho_a a^2}{16\pi^2 f^2}.$$  

(1.4)

Eq. 1.4 gives us an expression for the strain (potential) energy density in the system. Similar to a harmonic oscillator, the total energy in the system is equally divided between potential and kinetic energy (Lay and Wallace, 1995). Therefore, I
can express the total seismic energy density in the system as $\varepsilon_s = 2\varepsilon_d$, giving:

$$\varepsilon_s = \frac{\rho_a a^2}{8\pi^2 f^2}. \quad (1.5)$$

Eq. 1.5 thus provides us with a method for expressing the total seismic energy density $\varepsilon_s$ in terms of the maximum acceleration $a$ imparted to the rock. Therefore, equating $a$ with the gravitational acceleration $g_a$ on the surface of a spherical asteroid, and then multiplying $\varepsilon_s$ by the volume of this spherical asteroid gives us the total amount of seismic energy $E_s$ necessary to produce $1 g_a$ accelerations throughout the asteroid volume:

$$E_s = \frac{1\pi G^2 \rho_a^2 D_a^5}{108 f^2}, \quad (1.6)$$

where $G$ is the gravitational constant and $D_a$ is the asteroid diameter. Eq. 1.6 gives us the other half of our desired expression.

I now equate $E_i$ (Eq. 1.1) with $E_s$ (Eq. 1.6) and solve for the diameter of a stony projectile $D_p$ which meets this condition as a function of asteroid diameter $D_a$, yielding:

$$D_p = \left[ \frac{G^2 \rho_a^2 D_a^5}{9\eta \rho_p^2} \right]^{\frac{1}{2}}. \quad (1.7)$$

Fig. 1.6 shows a plot of Eq. 1.7, giving the minimum impactor diameter $D_p$ necessary to cause global seismic effects on an asteroid of diameter $D_a$ for three different seismic frequencies, using a typical asteroid impact speed of $v_p = 5 \text{ km s}^{-1}$ (Bottke et al., 1994), asteroid density of $\rho_a = 2700 \text{ kg m}^{-3}$ (near the Eros value of $2670 \pm 30 \text{ kg m}^{-3}$ (Yeomans et al., 2000)), projectile density of $\rho_p = 2500 \text{ kg m}^{-3}$, and impact seismic efficiency of $\eta = 10^{-4}$ (Schultz and Gault, 1975a; Melosh, 1989). Also shown is the minimum impactor diameter $D_p$ necessary to cause the disruption of an asteroid of diameter $D_a$, using the formulae given in Melosh and Ryan (1997) and Benz and Asphaug (1999), respectively. The region bounded by these two sets of curves (shown in gray) suggests the plausibility of a broad range of impactor sizes that can produce global seismic effects without disrupting the
target asteroid. For an asteroid the size of Eros (mean diameter $D_a \approx 17$ km) the minimum impactor diameter necessary to achieve global seismic accelerations of $1g_a$ is quite small, $D_p \approx 2$ m (0.5-10 m): far smaller than the size of impactor that would disrupt the asteroid: $D_p \approx 1.1$ km (Melosh and Ryan, 1997), $D_p \approx 1.6$ km (Benz and Asphaug, 1999). These plots are rather simplistic, however, because the important effect of seismic energy attenuation is not included. I will therefore revisit this analytical calculation further on in this study.

The impact seismic efficiency factor $\eta$, the fraction of the impactor’s kinetic energy that is ultimately converted to seismic energy within the target body, is an important (but poorly constrained) parameter. Various values from the literature include:

- Gault and Heitowitz (1963) report an upper limit of $10^{-2}$ for small laboratory impacts,
- Titley (1966) report $3 \times 10^{-1}$ to $3 \times 10^{-3}$ for nuclear and other large explosion sources,
- McGarr et al. (1969) report $10^{-4}$ to $10^{-6}$ from laboratory experiments,
- Latham and McDonald (1968) report $1 \times 10^{-5}$ to $5 \times 10^{-5}$ from White Sands missile impact tests,
- Latham and McDonald (1968) report $10^{-5}$ to $10^{-6}$ for low angle Lunar Module impacts on the Moon, and
- Schultz and Gault (1975a) estimate $10^{-3}$ to $10^{-5}$ from various sources.

As listed above, the Saturn IVB booster and Lunar Module (LM) impacts performed as part of the Apollo Passive Seismic Experiment (PSE) gave values of $\eta = 10^{-5}$-$10^{-6}$ (Latham and McDonald, 1968; Toksoz et al., 1974). However, only the long-period (LP) seismometers were used for these determinations, which had
Figure 1.6: (Lower curves) Minimum stony impactor diameter necessary to cause $1g_\theta$ accelerations throughout the volume of a stony asteroid of given diameter (destabilizing all regolith-covered slopes on the surface), for seismic frequencies $f$ of 1 Hz (dashed), 10 Hz (dot-dashed), and 100 Hz (dot-dot-dashed). (Upper solid curves) Minimum stony impactor diameter necessary to cause disruption of a stony asteroid of given diameter, calculated per Melosh and Ryan (1997) (top), and Benz and Asphaug (1999) (bottom). The region bounded by these curves, shown in gray, highlights the wide range of impactor sizes that can cause global seismic effects on an asteroid without disrupting it. This plot, however, does not include the important effect of seismic attenuation as the energy propagates throughout the asteroid volume.
an upper frequency limit of about 1 Hz (Toksoz et al., 1974). As will be shown in Ch. 2, the peak seismic frequencies produced by an impact are higher, generally falling within the range of 5-50 Hz. Consequently, the lunar impact determinations of $\eta$ sampled only the low-frequency, low-energy ‘wing’ of the impact seismic signal power spectrum, and I thus adopt $\eta = 10^{-6}$ as a loose lower limit on $\eta$ in our modeling. On the other hand, Schultz and Gault (1975a) adopted $\eta = 10^{-4}$ for large impacts, and this value is also given as a typical value for most impacts by Melosh (1989). I will therefore utilize two values for this constant in this work: (1) following previous precedent I adopt $\eta = 10^{-4}$ as my ‘typical’ value, and (2) I will also investigate the more restrictive case of $\eta = 10^{-6}$.

1.3 Determining the surface effects of seismic shaking

In Sec. 1.2 I demonstrated the availability of sufficient seismic energy from even modest-sized impactors for producing global seismic effects on an asteroid less than 200 km in diameter, but a much more detailed analysis is required to quantify this effect as a surface modification process. The precursor for this work is a classic study of the seismic effect of impacts on lunar surface topography, performed by Houston et al. (1973). Building upon these earlier techniques, I investigate the seismic modification process through three modeling phases: seismic modeling (Ch. 2), geomorphic modeling (Ch. 3), and impact cratering statistics modeling (Ch. 4 and Ch. 5). In Chapter 2, I develop a basic seismic theory for fractured asteroids, find the typical seismic frequencies generated by an impact, and then use these to synthesize generic impact seismograms for a test case asteroid (433 Eros). In Chapter 3, I compute the mechanical response of regolith-covered slopes to impact-induced seismic vibrations, and then apply this computed downslope flow to a morphological model of the degradation and erasure of impact craters. As an application, in Chapters 4 and 5 I use the crater degradation model in a stochastic cratering simulation which reproduces the statistics of the cratering record on the surface of 433 Eros.
and shows how seismic shakedown results in the observed deficit of small impact craters.

In the vein of future work, I also introduce the investigation of an additional effect of impacts on small bodies – the distribution of impact ejecta following impacts. In its present phase this investigation is being done as part of the Deep Impact mission, with the current focus of the modeling on the short-term ejecta curtain behavior. The work done for this mission is presented in Chapter 6. This work will eventually feed into the impact cratering statistics model (Chpts. 4 and 5) and replace the simplistic, lunar crater ejecta blanket estimations being used there.
CHAPTER 2

SEISMIC MODELING

2.1 Seismic energy dispersion in a fractured medium

The Galileo images of 951 Gaspra and 243 Ida revealed two battered objects, with visible systems of ridges and grooves on their surfaces, several large concavities (many presumed to be from impacts (Greenberg et al., 1994, 1996), and highly irregular shapes indicative of at least some degree of structural strength (Belton et al., 1992, 1996; Carr et al., 1994; Sullivan et al., 1996). Rather than being single stone monoliths or highly pulverized 'rubble piles,' these features suggest that these asteroids are something in between. Further work has characterized an entire spectrum of asteroid structural types, called 'gravitational aggregates' by Richardson et al. (2002), which span the extremes from monolith to highly comminuted rubble pile. Britt et al. (2002) identified a transition group in the central region of this spectrum, called 'fractured monoliths' or 'fractured asteroids,' and placed byt 951 Gaspra and 243 Ida into this transitional category based upon porosity estimates.

The NEAR-Shoemaker observations of 433 Eros likewise showed an asteroid that most likely falls into this transitional category of fractured monolith, based upon several lines of supporting evidence (see Figs. 2.1, 2.2):

- Indications of structural control of impact craters and other features (Prockter et al., 2002),

- The presence of a global network of visible joints, ridges, and grooves (Zuber et al., 2000; Prockter et al., 2002),
• A mean porosity measurement of about 20-30%, consistent with a fractured rock composition (Wilkison et al., 2002; Britt et al., 2002),

• A center-of-figure to center-of-mass distance of only 30-60 m; indicative of a relatively homogeneous structure, without large-scale heterogeneities. That is, the measured porosity is likely not due to huge void spaces (which would probably be asymmetrical in distribution), but is instead most likely due to a homogeneous fracture and/or pore-space distribution (Thomas et al., 2002),

• A highly irregular shape, indicative of some inherent structural strength (Thomas et al., 2002; Robinson et al., 2002).

While describing the geology of 243 Ida, Sullivan et al. (1996) suggested a likely similarity between the internal structure of a fractured S-type asteroid and the uppermost crustal layers of the Earth’s moon. Both are composed of silicate rock, presumably began as monolithic structures, and have since been exposed to impactor fluxes of similar power-law distribution for millions to billions of years—albeit of different overall magnitudes (Ivanov et al., 2002). This similarity should produce similar fracture structures within each, consisting of (see Fig. 2.3): (1) a thin comminuted regolith layer on the surface, (2) a highly fractured mixture of rock and regolith beneath (a ‘megaregolith’ layer), and (3) a decreasing gradient of fractured bedrock below (Sullivan et al., 1996). In the case of the upper lunar crust, this fracture structure extends to depths of about 20-25 km (Dainty et al., 1974; Toksoz et al., 1974), but in the case of asteroids the size of Gaspra, Eros, and Ida, this fracture structure should extend throughout the body.

This type of structure provides us with an advantage in modeling the seismicity of fractured asteroids, in that the seismic behavior of the upper lunar crust in response to impacts was well characterized during the Apollo era (see Fig. 2.4). These lunar seismic studies showed that the dispersion of seismic energy in a fractured, highly scattering medium is a diffusion process. Therefore, the seismic energy density $\epsilon_s$ at
Figure 2.1: An example of evidence for a joint and fracture structure underlying the regolith layer on 433 Eros, imaged by the NEAR-Shoemaker spacecraft, in the form of several structurally controlled, 'square' impact craters (MET 132151598, 218.91 W, 16.64 S, 5.57 m/pixel).
Figure 2.2: An example of evidence for a joint and fracture structure underlying the regolith layer on 433 Eros, imaged by the NEAR-Shoemaker spacecraft, in the form of a network of criss-crossing ridges and grooves, with a few, small, structurally controlled craters (MET 136266921, 218.72 W, 42.00 N, 4.58 m/pixel).
Figure 2.3: Schematic view of the analogy between the upper lunar crust (investigated via the Apollo seismic experiments) and a proposed fractured asteroid structure (Sullivan et al., 1996). If each began as monolithic rock, exposure to similar impactor populations should produce similar fracture structures within each: (1) a thin, comminuted regolith layer on the surface; (2) a highly fractured mixture of rock and regolith beneath (a ‘megaregolith’ layer); and (3) a decreasing gradient of fractured bedrock below. In the case of the upper lunar crust, this fracture structure extends to depths of about 20 – 25 km (Dainty et al., 1974; Toksoz et al., 1974), but in the case of asteroids, this fracture structure should extend throughout the body. Based on Fig. 4 of Dainty et al. (1974).
a given location within a fractured asteroid should obey the equation (Dainty et al., 1974; Toksoz et al., 1974):

$$\frac{\partial \epsilon_s}{\partial t} = K_s \nabla^2 \epsilon_s - \frac{2\pi f \epsilon_s}{Q},$$

(2.1)

where $t$ is the time, $K_s$ is the seismic diffusivity (in m$^2$ s$^{-1}$), and $Q$ is the seismic quality factor (seismic attenuation parameter).

I initially solve Eq. 2.1 in Cartesian coordinates for a rectangular target body of length $L$, width $W$, and height $H$. I also approximate the initial seismic energy distribution (injected by an impactor) as a delta function—reasonable because the impactor sizes considered here are much smaller than the target asteroid. Solving the resulting differential equation leads to the particular solution:

$$\epsilon_s(x, y, z, t) = e^{-\frac{2\pi ft}{Q}} \left[ 1 + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi x_o}{L} \cos \frac{n\pi x}{L} e^{-\frac{\kappa_n \pi^2 z^2}{L^2}} \right] \left[ 1 + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi y_o}{W} \cos \frac{n\pi y}{W} e^{-\frac{\kappa_n \pi^2 z^2}{W^2}} \right] \left[ 1 + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi z_o}{H} \cos \frac{n\pi z}{H} e^{-\frac{\kappa_n \pi^2 z^2}{H^2}} \right],$$

(2.2)

where the impact occurs at point $x_o, y_o, z_o$ (logically, a point on the surface should be chosen).

If the impact occurs at the origin (one corner of the target body) and the seismic receiver is placed at point $L, W, H$ (the opposite corner of the target body), and I further let the asteroid shape be cubical, such that $W = L$ and $H = L$ (where $L$ is the cubed root of the asteroid’s volume), Eq. 2.2 collapses to:

$$\epsilon_s(t) = e^{-\frac{2\pi ft}{Q}} \left[ 1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-\frac{\kappa_n \pi^2 z^2}{L^2}} \right]^3,$$

(2.3)

for the normalized energy density as a function of time, seen on one corner of the cube from an impact on the opposite corner. This is in good agreement with the two-dimensional solution described in Toksoz et al. (1974). The normalization scheme used here lets the seismic energy density $\epsilon_s$ go to 1.0 as time goes to infinity when no attenuation is present. Alternatively, the normalization can be performed by letting
Figure 2.4: Examples of one artificial and two natural impact seismograms recorded by the Apollo 12 seismic experiment (ALSEP) long-period (LP) instrument in 1969. Note the smooth, teardrop-shaped, seismic amplitude envelopes indicative of a diffusion process, and the long ‘coda’ tails (long duration vibrations) indicative of an extremely low seismic attenuation rate—primarily due to a near zero moisture content and vacuum conditions. Based on Fig. 2 of Latham et al. (1970).
the total injected seismic energy $E_s$ be equal to 1.0, by multiplying the right hand side of Eq. 2.2 by $\frac{1}{LWH}$ or the right hand side of Eq. 2.3 by $\frac{1}{L^*}$.

It may seem overly simplistic to use a cubical asteroid shape to represent an irregularly shaped asteroid such as Eros. My purpose here is to obtain a generic, globally representative, ‘far-side,’ seismic energy solution, using an impact-to-receiver distance of $\sqrt{3}L$: between that of the long-axis and short-axis lengths of the actual asteroid, but still having the correct asteroid volume into which the seismic energy is dispersed. I also obtained a spherical solution to Eq. 2.1, based upon the techniques outlined by Kreyszig (1993) and Arfken and Weber (2000):

$$
\epsilon_s(\theta, t) = \sum_{l=0}^{\infty} A_l P_l(\cos \theta) \sum_{n=0}^{\infty} B_l^n J_{s_{l+\frac{1}{2}}} (\alpha_l^n) e^{-\frac{\kappa (\alpha_l^n)^2 t}{R^2}},
$$

where the solution is the seismic energy density on the surface of a sphere of radius $R$, $\theta$ is the co-latitude, and the impact occurs at the pole of the sphere.

Solving for the various boundary conditions gives the following solutions for the constants in Eq. 2.4:

$$
A_l = \frac{2l + 1}{2} P_l(\cos \theta),
$$

$$
B_l^n = \frac{2 J_{s_{l+\frac{1}{2}}} (\beta_l^n)}{R (J_{s_{l+\frac{1}{2}}} (\beta_l^n))^2}.
$$

where:

$$
\alpha_l^n = \text{Roots of } \frac{\partial}{\partial r} J_{s_{l+\frac{1}{2}}}(r),
$$

$$
\beta_l^n = \text{Roots of } J_{s_{l+\frac{1}{2}}}(r),
$$

and both are taken at $r = R$ (the radius of the sphere). Tabulation tables for many of these roots are given in Abramowitz and Stegun (1965).

Note that this solution is much more difficult to implement (involving a sum over the roots of the derivatives of the spherical Bessel functions—a Neumann boundary condition) than my former Cartesian solution. I worked with this spherical solution enough to verify that it yields essentially the same result as the Cartesian solution.
when impact and receiver are placed on opposite points on the sphere (with a lot more work), such that the simpler Cartesian solution was adopted for use in this particular study. For future studies that look at the local seismic effects of impacts (different values of co-latitude \( \theta \)), the more complex spherical solution will be adopted. The goal of this study, however, is to characterize the less severe ‘global’ effects of seismic shaking, without looking at enhanced vibrations closer to an impact site or looking for focusing effects due to the asteroid’s irregular shape.

In Eqs. 2.2 and 2.3 the key propagation parameters are the seismic diffusivity \( K_s \) and the seismic quality factor \( Q \). The seismic diffusivity is defined as:

\[
K_s = \frac{1}{3} v_s l_s ,
\]

(2.9)

where \( v_s \) is the seismic P-wave velocity in competent rock, and \( l_s \) is the mean free path for the scattering of seismic waves; that is, the distance over which \( 1/e \) of the seismic energy is scattered. The seismic velocity \( v_s \) is determined by the rock’s elastic properties, and the mean free path for scattering \( l_s \) is directly proportional to the mean fracture spacing within the asteroid. To adopt reasonable assumptions for each, I use values consistent with the upper lunar crust (determined from the lunar seismic experiments): a competent rock seismic velocity of \( v_s = 3 \text{ km s}^{-1} \), and a range of mean free paths for scattering of \( l_s = 0.125-2.000 \text{ km} \) (Dainty et al., 1974; Toksoz et al., 1974).

The seismic quality factor \( Q \) deserves special attention because of its unusual history with regard to the lunar case. Initial analysis of Moon rocks returned by the Apollo astronauts showed rather typical values for \( Q \), on the order of a few hundred or so (Tittmann, 1977). At the same time, the lunar seismic experiments showed extremely high values for \( Q \) (Pandit and Tozer, 1970). The difference was reconciled by a series of experiments described in Tittmann (1977) and Tittmann et al. (1980)—among other papers by this group—which showed that under conditions of increasing vacuum and extremely low moisture content, many rocks (including the lunar samples) display an exponentially increasing seismic quality factor \( Q \) (decreasing seismic
attenuation) up to the values determined by the lunar seismic experiments. Because S-type asteroids, such as Eros, are likewise composed of silicate rock, reside in a vacuum, and have extremely low moisture contents, I adopt lunar-like seismic quality factors in my modeling. The values determined for the upper lunar crust fall into the range of \( Q = 3000-5000 \) with some hint of a frequency dependence found in the long-period (LP) instrument data (Dainty et al., 1974; Toksoz et al., 1974): such that this value may decrease somewhat with increasing seismic frequency. I therefore cautiously adopt values of \( Q = 1000-2000 \) in my modeling, due to the higher peak seismic frequencies produced by impacts than were typically detected by the Apollo LP seismic instruments (Dainty et al., 1974; Toksoz et al., 1974).

Applying these assumed values, Fig. 2.5 plots the seismic energy density as a function of time for (dotted) an impact and receiver located in the centers of perpendicular faces on a cubical target body (using Eq. 2.2) and (solid) an impact and receiver located on opposite corners of a cubical target body (using Eq. 2.3). These solutions employ an Eros volume of \( 2535 \pm 20 \text{ km}^3 \) (Robinson et al., 2002) which gives \( L = 13.64 \text{ km} \) and an opposing-corner, impact-to-receiver distance of \( \sqrt{3}L = 23.62 \text{ km} \). When compared to the actual mean diameter of Eros, \( 16.92 \pm 0.04 \text{ km} \) (Robinson et al., 2002), Eq. 2.3 thus yields energy profiles that are lower than what I would obtain using the actual mean diameter of the asteroid, giving us a reasonably conservative ‘far-side’ solution.

2.2 Minimum impactor size for global seismic effects (II)

An examination of Eq. 2.3 shows two exponential terms: one for the spatial propagation of seismic energy, \( e^{-\frac{Kz^2}{2z^2}} \), and one for the attenuation of seismic energy, \( e^{2\pi/\lambda} \). These two terms provide us with a means for adding an estimate for seismic energy propagation time and attenuation into my previous expression (Eq. 1.7) for the size of an impactor capable of producing global surface modification on a target asteroid.
Figure 2.5: A plot of the normalized seismic energy density for three minutes following an impact for (dotted) an impact and receiver located in the centers of perpendicular faces on a cubical target, using Eq. 2.2; and (solid) an impact and receiver located on opposite corners of a cubical target, using Eq. 2.3. Note the local seismic energy peak in the dotted curve as compared to the gentle build-up and decay of seismic energy shown in the solid curve.
An approximation for the amount of time necessary for seismic energy to propagate from one side of an asteroid to the other can be found by finding the time at which the spatial propagation term from Eq. 2.3 goes to $e^{-1}$ for the lowest spatial wave number $n$. That is, I find the ‘$e$-folding’ time constant for the expression, giving us a rough estimate of the time required for the first seismic waves from the impact to reach the far side of the asteroid. This yields:

$$t = \frac{L^2}{K_s \pi^2}. \tag{2.10}$$

Next, I substitute this time $t$ into the seismic energy attenuation term from Eq. 2.3, obtaining an estimate for the amount of seismic attenuation that occurs over the course of propagating from one side of the asteroid to the other. This gives:

$$A_s = e^{-\frac{2\pi L^2}{K_s \pi^2 Q}}. \tag{2.11}$$

where $A_s$ is my attenuation estimate.

I can now modify Eq. 1.1 to solve for the size of impactor necessary to produce 1 g$_g$ accelerations on the ‘far-side’ of a target asteroid, correcting for seismic attenuation. Multiplying Eq. 1.1 by $A_s$ gives:

$$E_i = \eta E_k = \frac{1}{12} \eta \pi \rho_p v_p^2 D_p^3 e^{-\frac{2\pi L^2}{K_s \pi^2 Q}}, \tag{2.12}$$

where $L$ has been replaced by the diameter of my target asteroid $D_a$.

Equating this new $E_i$ with the $E_s$ from Eq. 1.6 as before yields:

$$D_p = e^{\frac{2\pi L^2}{K_s \pi^2 Q}} \left[ \frac{G^2 \rho_p^3 D_a^5}{9 \eta \rho_p v_p^2 f^2} \right]^{\frac{1}{4}}. \tag{2.13}$$

Fig. 2.6 plots Eq. 2.13, showing the minimum impactor diameter $D_p$ necessary to cause global seismic effects on an asteroid of diameter $D_a$ for three different seismic frequencies, and using the same parameters listed in Sec. 1.2 along with a seismic diffusivity of $K_s = 0.250$ km$^2$ s$^{-1}$, and a seismic quality factor of $Q = 2000$. Note that these three frequencies represent the broad frequency spectrum present in a
single impact seismic signal, a feature which will be explored in detail in Sec. 2.3. As before, I also show the minimum impactor diameter $D_p$ necessary cause the disruption of an asteroid of diameter $D_a$, using the formulae given in Melosh and Ryan (1997) and Benz and Asphaug (1999). The region bounded by these two sets of curves (shown in gray) indicates that, although reduced as compared to Fig. 1.6, there remains a broad range of impactor sizes that can produce global seismic effects without disrupting small to medium sized asteroids. At large target asteroid sizes, however, the impactor size necessary to produce global seismic effects begins to approach the impactor size that can potentially disrupt the body. Consequently, a practical limit for global seismic effects exists, such that asteroids greater than about $\sim 100$ km diameter will experience only local seismic effects from individual impacts. Note also that the lower seismic frequencies suffer less attenuation and therefore penetrate farther than the higher seismic frequencies, a feature observed during the Apollo seismic experiments in that the short-period (SP) instruments were only able to detect relatively close impacts (both artificial and meteoritic), while the long-period (LP) instruments were able to detect impacts out to much greater ranges (Dainty et al., 1974; Duennebier and Sutton, 1974; Latham and McDonald, 1968; Toksoz et al., 1974).

2.3 The frequency spectrum of an impact

The goal of the seismic portion of this modeling work is to produce a series of generic, synthetic seismograms typical of impacts on a fractured asteroid. To use the previously derived seismic energy diffusion and attenuation theory to this end (Sec. 2.1), I first need to find the typical seismic frequency spectrum produced by a small asteroid impact (0.5 m to 500 m impactor diameter). However, with the single exception of Duennebier and Sutton (1974) for lunar impacts recorded via the Apollo 14 short-period (SP) PSE, there are no published power spectra for impact seismic signals. I therefore obtain my power spectra through detailed numerical simulation,
Figure 2.6: **(Lower curves)** Minimum stony impactor diameter necessary to cause 1 \( g_a \) accelerations throughout the volume of a stony asteroid of given diameter (destabilizing all regolith-covered slopes on the surface), for seismic frequencies \( f \) of 1 Hz (*dashed*), 10 Hz (*dot-dashed*), and 100 Hz (*dot-dot-dashed*). In this case, an estimate of seismic attenuation has been included (compare to Fig. 1.6), such that each seismic frequency has a finite distance over which it will be effective, with lower frequencies penetrating further than higher frequencies. Note that a single impact will produce a seismic frequency spectrum containing all of these frequencies (1-100 Hz). **(Upper solid curves)** Minimum stony impactor diameter necessary to cause disruption of a stony asteroid of given diameter, calculated per Melosh and Ryan (1997) (*top*), and Benz and Asphaug (1999) (*bottom*). The region bounded by these curves, shown in gray, continues to show a wide range of impactor sizes that can cause global seismic effects on small to medium sized asteroids without disrupting them. However, this analytical calculation does point to an upper asteroid size limit of about \( \sim 100 \text{ km} \) for global seismic effects from impacts. Larger asteroids will experience local seismic effects only.
using the *SALES-2* hydrocode package (Amsden et al., 1980; Collins et al., 2002) in its most basic mode: monitoring the elastic response of rock to an impact source, without viscosity or accumulated damage (Hooke’s law response only).

Rather than modeling an impact directly (with all of its inherent complexity and Lagrangian mesh stability problems), the impactor is instead treated as an idealized seismic signal source (Lay and Wallace, 1995; Walker and Huebner, 2004), having vertical and radial components in an axially-symmetric mesh (left half of Fig. 2.7). Two individual mesh elements are treated as dynamic point sources, with forces applied to them in the specified component directions. The total amount of energy injected into the mesh is proportional to the impact energy (through the seismic efficiency constant $\eta$), while the impulse velocity and duration are consistent with the contact-and-compression phase of a small impact (Melosh, 1989). The motion of the mesh in response to this brief impulse force is then monitored as a function of time to produce synthetic seismograms at various locations on the model’s outer free surface, as shown in the right half of Fig. 2.7 (*solid lines*). I tested this hydrocode seismic modeling method against a theoretical computation of the seismic response of a homogeneous half-space to an impact, using the techniques described in Kanamori and Given (1983) and Richards (1979). The resulting theoretical seismograms compare well with my numerical seismograms, and are shown in the right half of Fig. 2.7 (*dashed lines*).

Asteroids are, of course, not seismically homogeneous bodies, so the numerical modeling was next moved up to the simulation of an impact into a fractured, multiple material, spherical asteroid—about the size of Dactyl (1 km diameter). Small mesh and cell sizes are required to keep the frequency resolution of the mesh at 125 Hz in the slowest material (used to represent a fault gouge or regolith). This mesh setup is shown in Fig. 2.8 (A). The seismic motion of the free surface, shown in Fig. 2.8 (B) is recorded as a synthetic seismogram, from which the power spectrum is taken.

One important question regarding these hydrocode-produced seismograms is:
Figure 2.7: (Left **background**) This diagram illustrates the basic components of an impact seismic source, modeled in cylindrical coordinates as part of a hydrocode simulation. Nodal velocity vectors are shown during the initial acceleration of two selected mesh points, one downward and axial (\(z\) direction) and one radial and on the surface (\(r\) direction). (Left **foreground**) The resulting pressure contours in the hydrocode mesh after 0.04 sec. showing a hemispherical, expanding body (P) wave, and an advancing surface or Rayleigh (R) wave. (Right) Theoretical (dashed) and hydrocode produced (solid) surface seismograms at 0.5 km distance from an impact into a homogeneous rock half-space, showing a weak body (P) wave arrival at 0.25 sec. and a strong surface (Rayleigh) wave arrival at 0.5 sec. Based on Fig. 2 of Richardson et al. (2004).
Figure 2.8: (A) Cross-sectional view of an axially-symmetric hydrocode mesh showing the pressure contours produced by seismic waves propagating through a ‘fractured,’ 1-km-diameter, spherical, rock target following an impact. The wave propagation is a mix of unreflected, reflected, and multiply reflected wave-fronts, such that the propagation of seismic energy is beginning to approach the behavior of a diffusion process. (B) Hydrocode produced surface seismograms at 90° away (half-way around the spherical model surface) from an impact into this 'fractured,' 1-km-diameter, spherical, rock target. Note that although the vibrations are extremely mixed in nature, produced by multiple wave-front arrivals (both body and surface waves), an amplitude 'envelope' can be discerned in each, particularly in the vertical motion. Based on Fig. 2 of Richardson et al. (2004).
what is controlling the primary frequencies produced? To answer this question I performed additional (and more typical) hydrocode impact simulations for two different impactor sizes striking both a homogeneous spherical target and a fractured spherical target (as shown in Fig. 2.8 (A)). Fig. 2.9 shows the normalized vertical power spectra for a 4-m and 60-m impactor striking a 1-km-diameter homogeneous sphere (left column) and fractured sphere (right column) at 100 m s⁻¹, where the slow speed is necessary to maintain good stability in a Lagrangian mesh for at least 3 seconds following impact.

These slow-speed simulations indicate that the fracture blocks within the target act as a crude band-pass filter to the injected signal. That is, the fractured body preferentially passes those seismic frequencies close to the harmonic frequencies associated with the mean fracture spacing within the body. Note that even in the case of the larger impactor, which has an inherently lower injected frequency spectrum, the fractured mesh continues to select out the preferred 10-80 Hz range. I therefore choose a fracture structure that produces results consistent with impacts into the upper lunar crust. The resulting seismograms (Fig. 2.8 (B)) have a frequency spectrum generally falling between 1 and 100 Hz, with a peak at about 10-20 Hz. Other impact seismic studies have reported peak frequencies of 10-40 Hz (White Sands missile impacts (Latham and McDonald, 1968)), 20-30 Hz (Lunar Active Seismic Experiment (Houston et al., 1973)), and 5-15 Hz (Lunar Passive Seismic Experiment, short-period instrument (Duennebier and Sutton, 1974)).

With regard to my use of a two-dimensional hydrocode (as compared to a three-dimensional hydrocode), this frequency spectrum matching led us to a fracture spacing of 200-400 m in the mesh, a spacing that would necessarily be smaller in a three-dimensional mesh to achieve the same frequency results. The two-dimensional results, however, are adequate for my purposes since moving to the added complexity of a three-dimensional model would not add significantly to the power and phase spectra obtained. To be consistent with my selected fracture spacing, I additionally
Figure 2.9: **(Top row)** Normalized vertical power spectra (for seismic motion) from a 4-m impactor striking a 1-km-diameter, homogeneous rock sphere (*left*) and fractured rock sphere (*right*) at 100 m s\(^{-1}\). The slow speed helps to maintain stability in a Lagrangian mesh for at least 3 seconds. **(Bottom row)** Normalized vertical power spectra (for seismic motion) from a 60-m impactor striking a 1-km-diameter, homogeneous rock sphere (*left*) and fractured rock sphere (*right*) at 100 m s\(^{-1}\). The smaller impactor produces an inherently higher frequency spectrum than that produced by the larger impactor, when no fractures are present. However, when fractures are present these spectra show that the blocks within the fractured mesh act as a crude band-pass filter to the injected signal, preferentially passing those frequencies with wavelengths near the harmonics associated with the typical fracture spacing (about 10-80 Hz).
adopt a ‘nominal’ mean free path for seismic wave scattering of \( l_s = 0.250 \) km, although other values will be investigated (\( l_s = 0.125-2.000 \) km).

Also note that free oscillations of the entire, three-dimensional, irregularly shaped asteroid are not important to this study because the frequencies involved with such free oscillations will be out in the low-frequency, low-power ‘wing’ of the frequency spectrum produced by the impact. That is, the oscillation amplitudes necessary to create destabilizing accelerations on the asteroid surface will be long gone by the time the signal decays to the point that whole-body free oscillations dominate.

2.4 Creating synthetic impact seismograms

The diffusion theory for the propagation of seismic energy in a fractured asteroid from Sec. 2.1 can be combined with the power spectra of an impact seismic signal from Sec. 2.3 to produce synthetic seismograms for a variety of impactor sizes on a given model asteroid target. To synthesize a generic seismogram for a body having the volume and approximate mean diameter of Eros, I begin by finding the fraction of impactor energy that is converted to seismic energy, and then divide this energy into frequency components in accordance with the power spectra obtained in Sec. 2.3. Stepping through time, Eq. 2.3 is then used to simulate the build-up of seismic energy by diffusion and the loss of seismic energy by attenuation for each frequency component. These components are then recombined using inverse Fourier analysis to produce a final seismogram level at each time step.

Fig. 2.10 shows the ‘far-side’ seismic vibrations resulting from the impact of a 10-meter stony object into my Eros-like model asteroid, which would produce an \( \sim 300 \) m impact crater on the surface (compare this seismogram to those in Fig. 2.4). Note that accelerations exceeding the surface gravity of the asteroid (taken to be 5 mm s\(^{-2}\)) last for about five minutes following the impact. I produced similar generic, synthetic seismograms for a wide range impactor diameters (from 0.5 m to
500 m) and my Eros test-case target asteroid. This modeling shows that seismic reverberations exceeding the surface gravity of the asteroid last from a few minutes for impactors of a few meters in diameter, up to about an hour for the largest impactor sizes modeled (a few hundred meters in diameter). The next stage in this study is to investigate how these impact-induced seismic motions will affect the surface morphology of the asteroid. In particular, does this seismic activity cause the degradation of impact craters?
Figure 2.10: (A) The first 6 minutes of a synthetic seismogram for the ‘far-side’ of Eros following the strike of a 10-m stony impactor, showing an asymmetrical, mixed-phase, reverberation signal (compare to Fig. 2.4). (B) The corresponding seismic accelerations (gray) for the seismogram shown in (A). The two dashed lines indicate the approximate surface gravity magnitude ($g_a = 5 \text{ mm s}^{-2}$), indicating that seismic accelerations that exceed 1 $g_a$ last for about 5 minutes following this impact. Based on Fig. 2 of Richardson et al. (2004).
CHAPTER 3

GEOMORPHIC MODELING

3.1 Downslope motion modeling

To evaluate the effects of seismic shaking on asteroid surface morphology, a downslope motion model was developed which takes the accelerations recorded in the synthetic seismograms described in Ch. 2.4 and applies them to a hypothetical regolith layer resting on a slope of variable angle, and placed in an asteroid gravity field. For this, I use a form of Newmark slide-block analysis (Newmark, 1965), which can be applied when the regolith layer thickness under consideration is much smaller than the seismic wavelengths involved (Houston et al., 1973; Jibson et al., 1998). This assumption works well for all but the highest impact seismic frequencies, which are the most quickly lost by attenuation. Under this restriction, I can approximate the motion of a mobilized regolith layer by modeling the motion of a rigid block resting on an inclined plane (for discussions of forces involved see Lambe and Whitman (1979); Newmark (1965); Melosh (1979)). I compute the accelerations imparted to the block by the asteroid’s surface gravity (static loading) and seismically shaking slope (dynamic loading) to obtain an overall block (layer) displacement. See Appendix A for a description of the code used to produce this simulation. Fig. 3.1 shows a schematic view of this model and an example of the type of layer motions achieved during seismic shaking. Note that much of the downslope motion observed is a combination of hopping and sliding, which is much more effective at moving regolith downslope than simple stick-slip motion (horizontal sliding only). Figs. 3.2 and 3.3 show a test of this model as compared to a classic shake-table laboratory experiment, in which the downslope motion recorded by the shake-table (Goodman
and Seed, 1966; Lay and Wallace, 1995) is accurately reproduced by the numerical slide-block model.

Two types of regolith layers were tested: (1) a Mohr-Coulomb, non-cohesive, uniform porosity, sand-like layer; and (2) a regolith layer having a porosity and cohesion gradient as a function of depth. This second model regolith type stems from the evidence on Eros for weak regolith cohesion (non-zero shear strength), most notably in the form of steep crater walls in small craters which were clearly formed in regolith and ponded deposits (Robinson et al., 2001). On the other hand, broader slope studies indicate that on 300-m scales, only 1-3% of slopes are above typical angles of repose (about 30°), with an observed maximum of 36° (Thomas et al., 2002). Thus, although some features do show indications of cohesion and strength (in the form of crater walls, boulders, outcrops, ridges, and groove edges), areas of obvious regolith coverage all tend to lie below typical angles of repose (Thomas et al., 2002; Robinson et al., 2002). There is also ubiquitous evidence of slope destabilization and the downslope migration of regolith over the entire surface of the asteroid, as mentioned in Sec. 1.1. This evidence suggests that although present, existing cohesion forces seem to be relatively easy to overcome.

The importance of even weak cohesive forces in this ‘milli-gravity’ environment can be demonstrated by looking at the Factor of Safety equation for my slide-block model:

\[
FOS = \frac{C + (\mu_s \rho_r g_s h \cos \theta)}{\rho_r g_s h \sin \theta}
\]  

(3.1)

where \(FOS\) is the factor of safety, \(C\) is the cohesion, \(\mu_s\) is the coefficient of static friction, \(\rho_r\) is the regolith depth, and \(\theta\) is the slope angle. Under the gravity conditions on the surface or Eros, a typical dry-sand regolith layer 5 meters in depth with a low cohesion value of \(C = 100\) Pa can maintain angles of stability (\(FOS = 1\)) up to 80°. I therefore felt it necessary to investigate the effects of cohesion in my slide-block modeling. Lacking any actual data on the asteroid’s regolith beyond the evidence provided by the NEAR images, I build a model regolith by (again) relying
Figure 3.1: (Left) A basic illustration of the Newmark slide-block model. The regolith layer resting on a slope is represented by a rigid block resting on an inclined plane. Forces on the rigid block include surface gravity (static loading), seismic accelerations (dynamic loading—applied by the inclined plane), and frictional forces (both static and dynamic). Ballistic launching of the block (layer) is also permitted and tracked in this model. (Upper right) Overall motion of an asteroid regolith layer (1 m depth) resting on a 10° slope, under the seismic shaking conditions produced by a 10-m impactor on the ‘far-side’ of an Eros-like asteroid. Note that the motion involves vertical hopping in addition to horizontal sliding in the asteroid’s very low gravity field (~ 5 mm s^{-2}). (Lower right) Close up view of the vertical motion of the inclined plane (dotted) and slide-block (solid), showing the vertical launching and ‘flight’ of the block (regolith layer) in detail.
Figure 3.2: The numerically-modeled motion of a layer of dry, loose sand resting on a slope of 30°, under the dynamical loading produced by horizontal seismic shaking with a frequency of 7 Hz and maximum accelerations of about 0.4 \( g_{\text{earth}} \). Over the course of 1 second, the 1 meter deep regolith layer moves downslope about 45 mm.
Figure 3.3: The experimentally measured motion of a layer of dry, loose sand resting on a slope of $31^\circ$, under the dynamical loading produced by horizontal seismic shaking with a frequency of about 7 Hz and maximum accelerations of about 0.4 $g_{earth}$. Over the course of 1 second, the sand layer moves downslope about 45 mm (experiment described in Goodman and Seed (1966) and Lay and Wallace (1995)).
on the data set collected during the Apollo era on the lunar regolith as my starting point and best analogy.

Analytical calculations of slope factors of safety, angles of stability, and angles of repose indicate that directly applying lunar regolith properties (taken from Houston et al. (1973)) in an Eros gravity field results in a regolith layer that can maintain nearly vertical (greater than 85°) slopes up to depths of 20 meters. Thus, using the lunar regolith values directly results in a regolith layer which contradicts the visible evidence from Eros, where most regolith slopes lie at below typical values (< 30°) for the angle of repose (Thomas et al., 2002; Robinson et al., 2002). In an effort to bring the model more in-line with the actual evidence, I therefore reduced the cohesion formula derived for the lunar regolith by an order of magnitude, to obtain the following terms for a hypothetical asteroid regolith:

\[
P_r = 0.52 - 0.007h, \quad (3.2)
\]
\[
V_r = \frac{P_r}{1 - P_r}, \quad (3.3)
\]
\[
C_r = 9.82 \times 10^{1.14 - \frac{0.7}{V_r}}, \quad (3.4)
\]
\[
\phi_r = \tan^{-1} \frac{0.7}{V_r}, \quad (3.5)
\]

where \(P_r\) is the regolith porosity in %, \(h\) is the mobilized regolith layer thickness (depth), \(V_r\) is the void ratio, \(C_r\) is the cohesion in Pascal (N/m²), and \(\phi_r\) is the angle of internal friction.

These formulae give values of regolith cohesion \(C_r\) of 13-146 Pa for the first 20 meters of regolith depth, which is sufficient to permit angles of stability of > 70° for shallow depths (up to a few meters), but also gives angles of repose of ~ 35°-45° for regolith depths up to 20 meters. This large difference between the regolith angle of stability and repose is at least somewhat consistent with the observations described by Robinson et al. (2001), without violating the larger scale regolith properties described by Thomas et al. (2002). My hypothetical model regolith is obviously not much more than a guess at the true Eros regolith properties, but does provide us
with a way of testing seismic shaking against both a simple non-cohesive regolith and a first-order estimate of a cohesive asteroid regolith.

3.2 Minimum impactor size for global seismic effects (III)

One way of testing my combination of synthetic seismograms and downslope regolith flow modeling is to compare these numerical methods to my previous analytical calculations of the minimum size of impactors necessary to cause global seismic surface effects. Figs. 3.4 and 3.5 map out the diameter of impactor necessary to produce greater than $1g_a$ accelerations (vertical launching) of a $h = 1$ m thick model regolith layer resting on a slight $2^\circ$ slope, for a variety of asteroid diameters, seismic properties, and two different regolith types. Fig. 3.4 uses ‘nominal’ seismic propagation conditions ($\eta = 10^{-4}$, $Q = 2000$) and a non-cohesive regolith layer, while Fig. 3.5 uses more restrictive seismic propagation conditions ($\eta = 10^{-6}$, $Q = 1000$) and the cohesive regolith model described in Sec. 3.1. In both cases, a variety of seismic diffusivities $K_s$ values are tested: from $K_s = 0.125$ km$^2$ s$^{-1}$ to $K_s = 2.000$ km$^2$ s$^{-1}$, corresponding to mean free paths for scattering of $l_s = 0.125$-2.000 km.

This numerical modeling matches the previous analytical result (shown in Fig. 2.6) quite well and also shows that the mechanism continues to be effective even under the more restrictive conditions used in the second case, Fig. 3.5. These results also support a practical limit for impact-induced global seismic effects, such that asteroids greater than about 70-100 km diameter will experience only local seismic effects from all but the largest impacts, as previously described in Sec. 2.2.

With regard to my Eros test case, this slide-block modeling indicates that for an asteroid having the volume and approximate mean diameter of Eros, global downslope motion on all slopes ($2^\circ$-$30^\circ$) and for $l_s = 0.250$ km begins at impactor diameters of $\sim 2$ m for a non-cohesive regolith and ‘nominal’ seismic conditions, and $\sim 40$ m for a simple cohesive regolith and more severe seismic propagation conditions. These values are still far smaller than the size of impactor that would
Figure 3.4: (Lower curves) Plots of the minimum size of impactor necessary to produce greater than $1g_a$ accelerations (vertical launching) of an $h = 1$ m thick model regolith layer resting on a slight $2^\circ$ slope, for a variety of asteroid sizes, seismic properties, and regolith types. In this simulation, I used ‘nominal’ seismic propagation conditions ($\eta = 10^{-4}$, $v_s = 3$ km s$^{-1}$, $Q = 2000$) and a non-cohesive regolith layer. A variety of seismic diffusivity values are tested, from $K_s = 0.125$ km$^2$ s$^{-1}$ to $K_s = 2.000$ km$^2$ s$^{-1}$, corresponding to mean free paths for scattering $l_s$ of: 2.000 km (solid), 1.000 km (dashed), 0.500 km (dot-dashed), 0.250 km (dot-dot-dashed), and 0.125 km (dot-dot-dot-dashed). (Upper solid curves) Minimum stony impactor diameter necessary to cause disruption of a stony asteroid of given diameter, calculated per Melosh and Ryan (1997) (top), and Benz and Asphaug (1999) (bottom). The region where global surface effects can occur from a single impact without disrupting the asteroid is shown in gray.
Figure 3.5: (Lower curves) Plots of the minimum size of impactor necessary to produce greater than $1g_a$ accelerations (vertical launching) of an $h = 1 \text{ m}$ thick model regolith layer resting on a slight $2^\circ$ slope, for a variety of asteroid sizes, seismic properties, and regolith types. In this simulation, I use a more restrictive seismic propagation conditions ($\eta = 10^{-6}$, $v_s = 3 \text{ km s}^{-1}$, $Q = 1000$) and the cohesive regolith model described in Sec. 3.1. A variety of seismic diffusivity values are tested, from $K_s = 0.125 \text{ km}^2 \text{ s}^{-1}$ to $K_s = 2.000 \text{ km}^2 \text{ s}^{-1}$, corresponding to mean free paths for scattering $l_s$ of: $2.000 \text{ km}$ (solid), $1.000 \text{ km}$ (dashed), $0.500 \text{ km}$ (dot-dashed), $0.250 \text{ km}$ (dot-dot-dashed), and $0.125 \text{ km}$ (dot-dot-dot-dashed). (Upper solid curves) Minimum stony impactor diameter necessary to cause disruption of a stony asteroid of given diameter, calculated per Melosh and Ryan (1997) (top), and Benz and Asphaug (1999) (bottom). The region where global surface effects can occur from a single impact without disrupting the asteroid is shown in gray.
disrupt the asteroid, and show the availability of a wide range of impactor sizes capable of triggering the more obvious indications of downslope regolith motion observed on the surface of Eros (shown in Figs. 1.1, 1.2, 1.3, and 1.4).

3.3 Crater degradation by topographic diffusion

The next phase of the study is to demonstrate how seismically triggered downslope flow also contributes to the erosion of cratered topography. I begin by applying my Eros test-case seismograms to a Newmark slide-block model of an \( h = 1 \) m non-cohesive regolith layer resting on a slope, done over a wide variety of impactor sizes (0.5 m - 500 m) and slope angles (2° - 30°). This modeling yields a set of curves plotting downslope volumetric debris flux per impact \( q_i \) (m\(^3\) m\(^{-2}\)) as a function of slope \( \nabla z \) (Fig. 3.6 (A)), which shows one curve for each impactor size. The motion displayed in these curves is typical of non-linear, disturbance-driven downslope debris flow, and can be described by the equation (Roering et al., 1999):

\[
q_i = \frac{K_i \nabla z}{1 - \left[\frac{\nabla z}{S_c}\right]^2}.
\] (3.6)

I therefore use Eq. 3.6 to fit each downslope flux curve and determine the downslope diffusion constant \( K_i \) (Fig. 3.6 (B) solid circles), and the critical slope \( S_c \). Note that under normal sliding conditions, the critical slope \( S_c \) would be equal to the coefficient of static friction assigned to the regolith layer in the model, \( \mu_s = 0.7 \), but due to the ballistic hopping of the layer while in motion, this parameter was better left as a free parameter in the fit, with values of \( S_c \) falling between 0.55-0.65 (near the assigned coefficient of dynamic friction, \( \mu_r = 0.6 \)). These fits yield a set of diffusion constants \( K_i \) (m\(^3\) m\(^{-2}\) per impact) as a function of impactor diameter \( D_p \) that follow power-law relationships (see Fig. 3.6 (B)). These relationships fall into two regimes: \( D_p = 1-4 \) m, where sliding occurs in stick-slip fashion; and \( D_p > 4 \) m, where sliding occurs in hop-slip fashion. These computed diffusion constant values can be thought of as a ‘maximum’ downslope regolith displacement per impact (on
Figure 3.6: (A) Newmark slide-block model results for six impactor sizes, plotting volumetric flux per impact \( q_i \) (m\(^3\) m\(^{-2}\)) as a function of slope gradient \( \nabla z \) and displaying the non-linear relationship typical of disturbance-driven flow (Roering et al., 1999). (B) Downslope diffusion constants per impact \( K_i \) (m\(^3\) m\(^{-2}\) or simply \( m \)) plotted as a function of impactor size \( D_p \), where solid circles show the derived values from Eq. 3.6 and solid lines show a linear least-squares fit to these points. The resulting power-law relationships fall into two regimes: \( D_p = 1-4 \) m, where sliding occurs in stick-slip fashion; and \( D_p > 4 \) m, where sliding occurs in hop-slip fashion. This plot is compared with the seismic ‘jump’ distances reported in Greenberg et al. (1994, 1996), which were estimated from surface velocities on a homogeneous, spherical hydrocode model following impact. Based on Fig. 3 of Richardson et al. (2004).
very steep slopes), and as such permit us to perform a rough comparison between these $K_i$ values and the regolith seismic ‘jump’ distances (per impact) reported in Greenberg et al. (1994, 1996). Shown in Fig. 3.6 (B), these ballistic ‘jump’ distances were estimated from the maximum velocities imparted to the surface of a homogeneous, spherical hydrocode model following impact, and therefore represent the downslope motion resulting from a single seismic ‘jolt’ (P-wave and Rayleigh wave passage). Not surprisingly, my downslope diffusion constants are significantly greater (by a order of magnitude or more) because in this study I have modeled the amount of downslope motion resulting from the entire duration of seismic motion following an impact, which can last from a few minutes to up to an hour. Typical values for duration of downslope motion following an impact are shown in Fig. 3.7.

Note that if the slope $\nabla z$ is small, Eq. 3.6 becomes approximately linear with respect to slope:

$$q_i \approx K_i \nabla z,$$

which will permit the computed values of downslope diffusion per impact $K_i$ to be used in a topographic modification model of the degradation and erasure of impact craters. This modeling is done using an analytical theory of erosion for transport-limited downslope debris flow first described by Culling (1960). That is, I assume that the downslope flow of regolith is controlled by the transportation rate and not by the regolith supply or production rate (weathering-limited flow) (Nash, 1980): an assumption which should hold true for most of the surface of Eros due to its copious regolith coverage.

I begin with an expression for the conservation of mass on an infinitely small portion of a hillslope, in Cartesian coordinates (Culling, 1960):

$$\frac{\partial z}{\partial t} = - \left[ \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} \right],$$

where $z$ is the elevation, and $f_x$ and $f_y$ are the flow rates of regolith in the $x$ and $y$ directions, respectively.
Figure 3.7: The duration of seismic reverberations on an Eros-sized asteroid which are capable of causing a dry, loose (non-cohesive) regolith to migrate down a 12° slope, following a 5 km sec$^{-1}$ impact by a stony body of diameter $D_p$. Significant seismic vibrations (those that affect the surface morphology) can last for a few minutes to up to an hour following a large impact.
If the regolith layer is isotropic with regard to material flow, and the flow rate is linearly proportional to the slope’s gradient (as in Eq. 3.7), then:

\[ f_x = -K_d \frac{\partial z}{\partial x}, \quad \text{and} \quad f_y = -K_d \frac{\partial z}{\partial y}, \quad (3.9) \]

where \( K_d \) is a downslope diffusion constant, which has units of \( \text{m}^3 \text{ m}^{-2} \text{ s}^{-1} \) (volume flux per unit time) or \( \text{m} \text{ s}^{-1} \) (downslope motion per unit time).

Substituting Eq. 3.9 into Eq. 3.8 gives:

\[ \frac{\partial z}{\partial t} = K_d \left[ \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right], \quad (3.10) \]

which I re-write in diffusion equation form:

\[ \frac{\partial z}{\partial t} = K_d \nabla^2 z. \quad (3.11) \]

Because I wish to investigate the degradation of a fresh impact crater due to the infilling of regolith (through downslope flow), I take advantage of the axial-symmetry of the crater’s shape and solve this diffusion equation in cylindrical coordinates \((r, z)\). The general solution is given by (based upon the techniques outlined in Kreyzig (1993)):

\[ z(r, t) = \sum_{k=0}^{\infty} C_k J_o(kr) e^{-K_d k^2 t}, \quad (3.12) \]

where \( h \) is the mobilized regolith layer thickness, \( k \) is the spatial wave number, and \( J_o \) is a zeroth order Bessel function of the first kind. Note that this equation is a function of time, applicable when the diffusion constant \( K_d \) is some constant rate with respect to time (units of either \( \text{m}^3 \text{ m}^{-2} \text{ s}^{-1} \) or \( \text{m} \text{ s}^{-1} \)). In my case, however, I have determined a specific amount of downslope diffusion per impact \( K_i \) (units of \( \text{m}^3 \text{ m}^{-2} \) per impact, or simply \( \text{m} \) per impact), which is itself a function of the impactor’s size. Thus, I desire a solution that becomes a function of the number of impacts (and their individual diffusion constants).

Converting to the appropriate variables gives:

\[ z(r, t) = \sum_{k=0}^{\infty} C_k J_o(kr) e^{-Khk^2}, \quad \text{and} \quad (3.13) \]
where:

\[ K = \sum_{i=0}^{n} K_i, \quad (3.14) \]

and \( n \) is the number of impacts and \( K_i \) is the amount of downslope diffusion per impact.

The constant terms, \( C_k \), are computed from the initial topographic form, for which I use a fresh impact crater shape taken from Melosh (1989);

\[ C_k = -\frac{dD^2}{128} k^2 D^2 e^{-\frac{k^2 D^2}{48}} \quad (3.15) \]

where \( d \) is the crater depth, \( D \) is the crater diameter, and where the crater has an initial \( d/D \) ratio of 0.2. Of particular note, the exponential term in Eq. 3.15 produces a Gaussian distribution of \( C_k \) values with a peak amplitude at wave number \( k_o = 4/D \) (See Fig. 3.8).

This solution is used to simulate the degradation (filling) of a crater due to seismic shaking (shown in Figs. 3.9, 3.10), given my derived downslope diffusion constants (I have termed this ‘seismic shakedown’). The most important term in this solution is the relaxation term, \( R = e^{-Khk^2} \), and in my modeling the crater is considered to be erased after six \( 1/e \) decays, \( R = e^{-6} = 0.0025 \), which gives a depth to diameter \( (d/D) \) ratio of 0.0005 when \( R = e^{-6} \) is (artificially) assumed over all spatial wave numbers. In actuality, the amount of relaxation for a given amount of downslope regolith diffusion \( K \) is strongly dependent upon the wave number \( k \), with higher wave numbers being more strongly affected than lower wave numbers. Thus, sharp crater rims and other high frequency features will be the first to relax (erode), while the overall bowl shape of the crater will be the last.

Since my Bessel function form of the initial crater shape consists of a narrow Gaussian range of spatial wave numbers (See Fig. 3.8), the point at which a crater becomes erased can be approximated by substituting \( k_o \) for \( k \) in the relaxation term \( R \) and equating the arguments \(-6 = -K h \left[ \frac{4}{D} \right]^2 \) to give:

\[ K \geq \frac{3D^2}{8h}, \quad (3.16) \]
Figure 3.8: A plot of the constant term $C_k$ as a function of wave number $k$ using Eq. 3.15 and computed for an impact crater 200 meters in diameter. In this instance the principal wave number $k_o = 0.02 \text{ m}^{-1}$, for a fundamental wavelength of 50 m.
Figure 3.9: Vertical cross-sections taken through a 200-m-diameter crater (shown in *gray*), plotted at four different times and showing its gradual degradation and erasure by impact-induced seismic shakedown on an asteroid having the same volume and surface area as Eros. Complete erasure occurs at a crater age of about 30 Myr in a Main Belt impactor flux. A 20-m-crater under the same conditions, will have a lifetime of about 300 kyr. Note the rapid initial degradation while the slopes are still relatively high, followed by a more gradual degradation as slopes flatten.
Figure 3.10: A field of softened and degraded craters on the surface of Eros, showing a range of morphologies consistent with degradation by seismic shakedown. Compare these morphological forms to Fig. 3.9 (MET 138807458, 154.34 W, 6.15 S, 5.12 m/pix)
for the erased state. That is, I find the total amount of downslope diffusion that takes the relaxation term $R$ to $e^{-6}$ for the principle wave number of the crater. When this amount of downslope diffusion $K$ (given by Eq. 3.16) is applied to Eq. 3.13 for all spatial wave numbers, I obtain a depth to diameter ($d/D$) ratio of 0.0041; at least twice as flat as what could reasonably be detected from NEAR images, $d/D = 0.01$ (Robinson et al., 2002). Eq. 3.16 therefore permits an assessment of an impact crater’s ‘seismic damage,’ as downslope diffusion accumulates over time and subsequent impacts (via Eq. 3.14) until final crater erasure. This expression will be used in the following chapter to add the effect of seismic damage to a model of the evolution of the cratering record on the surface of an asteroid.
CHAPTER 4

IMPACT CRATERING STATISTICS MODELING

4.1 A stochastic cratering model

In the final phase of this study, I use these seismic and geomorphic results to model the evolution of the crater size-frequency distribution on a simulated asteroid surface, and show how seismic modification changes the crater population statistics. This model uses Monte Carlo techniques (Press et al., 1992) to populate a two-dimensional continuous test surface, 34 km × 34 km in size (1156 km², near the Eros value of 1125 km² (Robinson et al., 2002)), with craters as a function of time. The simulation then allows these craters to be obliterated by the effects of subsequent impacts: (1) erosion by superposing craters, (2) blanketing by impact ejecta, and (3) erasure by seismic shakedown. The following subsections describe the details of the model and discuss applications of this model to various end-member impactor populations.

4.1.1 Details of the Monte Carlo cratering model

As mentioned above, the stochastic cratering model uses Monte Carlo techniques (Press et al., 1992) to populate a model surface with craters as a function of time and then allows them to be eroding by superposing craters, covered with impact ejecta, and seismically shaken by subsequent impacts. The model is loosely based upon similar, previous models (Woronow, 1978; Chapman and McKinnon, 1986) and consists of six two-dimensional matrix layers (representing the surface area of an asteroid) to form a ‘pseudo three-dimensional’ model. Two layers are used to store crater diameter values, two are used to store ejecta coverage values, and two
are used to store crater seismic damage values. The purpose for having two sets of information is to permit the superposition of smaller craters on top of larger craters, while preserving the information about the larger crater (below). The superposing and other crater erasure rules are explained in the following sections.

The computer program begins by setting up a blank asteroid surface area for study, consisting of a six-layer 1700 × 1700 element matrix, where each element represents a 20 m × 20 m pixel on a 34 km × 34 km surface area (1156 km², close to the Eros surface area of 1125 km² (Robinson et al., 2002)). All six matrix layers (crater diameters, ejecta coverage, and seismic damage) begin at zero values. The program next reads in a supplied data file which gives the probability of an impact from a particular sized impactor per year per square kilometer on the surface of an asteroid in the Main Belt, based on the population derived by O’Brien and Greenberg (2005). It then uses a random number generator and the supplied probabilities to produce a series of random impactor sizes and impact locations on the surface. Crater diameters are calculated from the impactor size, and each new crater is placed on the surface by writing the craters diameter value to all pixel elements within the crater’s radius, in accordance with the superposition rules described below. Also, within the new craters radius, all ejecta coverage and seismic damage values are set to zero. Note that the model matrix has periodic boundary conditions, such that craters positioned near a border (either vertical or horizontal) are wrapped around to the opposite side of the matrix. In effect, this creates a smooth, continuous surface area for study, without boundaries and boundary effects on the crater counting.

4.1.2 The Main Belt impactor population

The impactor population used by this model was selected to represent the average population of asteroids present in the Main Belt, where Eros has spent most of its lifetime (Michel et al., 1998). Fig. 4.1 plots the numerical distribution of asteroid sizes in the Main Belt, as determined from a variety of observational surveys and
model estimates. For my modeling purpose, I desired an impactor population in the size range from 0.5-500 m, which is below the sizes of asteroid that can be surveyed directly (also shown in Fig. 4.1). I therefore had to rely upon model estimates for the distribution of smaller asteroids (impactors) in the Main Belt, from 0.5-500 m in size.

The impactor population that I chose for use in my simulations was produced via the asteroid collision and dynamic population modeling performed by David O'Brien and described in O'Brien and Greenberg (2005). O'Brien and Greenberg's asteroid population was derived so as to be consistent with six constraints:

1. the observed population of the Main Belt asteroids from various surveys,
2. the observed population of the Near-Earth asteroids from various surveys,
3. the observed number of asteroid families in the main belt,
4. the measured cosmic ray exposure ages of meteorites,
5. the preservation of the basaltic crust of Vesta, and
6. the observed cratering records on asteroids.

In their model, O'Brien and Greenberg (2005) also includes the Yarkovsky effect as a means of removing small asteroids from the Main Belt and delivering them to the near-Earth population (for a discussion on this effect, see (Farinella et al., 1998)). Even with this effect added, the cumulative log-log slope of the small Main Belt asteroid population remains relatively steep at $\sim -3$, with no deficit of small impactors indicated by the model (a deficit suggested by Chapman et al. (2002) to potentially explain the lack of small craters on Eros).

Fig. 4.2 shows an expanded view of the numerical distribution of impactor produced by the O'Brien and Greenberg (2005) model, as it compares with similar impactor population models used by Belton et al. (1992) and Greenberg et al. (1994,
Figure 4.1: Cumulative distribution curves showing estimates for the number of asteroids in the Main Belt based upon a variety of telescopic surveys (listed in the legend), which extend down to diameters of about 500 m (0.5 km). Also shown are model estimates for the number of asteroids in the Main Belt, taken from O’Brien and Greenberg (2005) (solid) and Greenberg et al. (1994, 1996) (dashed), which extend down to diameters of 1 m (0.001 km). Figure taken from O’Brien and Greenberg (2005).
1996) to model the cratering records on the asteroids 951 Gaspra and 243 Ida. In this figure, the vertical axis has been converted from a cumulative number of asteroids (as shown in Fig. 4.1) to a cumulative number of impacts per year per square kilometer of surface area on the target body, in accordance with the asteroid collision probability estimates described in Bottke and Greenberg (1993).

4.1.3 Impactor to crater diameter relationship

The relationship between impactor size and resulting crater size on an asteroid-sized body is a subject of continuing study by various researchers. For my modeling, I accept the consensus that crater sizes on these bodies will generally be in the strength-scaling regime at small sizes and transition to the gravity-scaling regime at larger crater sizes, consistent with the numerical hydrocode modeling described in Greenberg et al. (1994), Greenberg et al. (1996), and Nolan et al. (1996). These studies led us to the adopt a simple ‘cube-root’ scaling-law for Eros such that the resulting crater sizes are directly proportional to the impactor size (Melosh, 1989):

\[ D = 30D_p. \]  \hspace{1cm} (4.1)

This scaling-law is plotted in Fig. 4.3, showing it in comparison to strength scaling (Holsapple, 1993), gravity scaling (Holsapple, 1993; Melosh, 1989), and the hydrocode simulations for 951 Gaspra and 243 Ida (Greenberg et al., 1994, 1996).

4.1.4 Erasure by crater superposing

Two matrix layers are maintained to store the crater diameter values: a ‘first generation’ layer and a ‘second generation’ layer. The first generation layer is generally the more permanent layer, storing information about larger and longer lasting craters. The second generation surface is generally the more temporary layer, particularly for small craters superposed upon a larger crater but not contributing to the erasure of the larger crater. Initially, new craters are written onto both 1st and 2nd generation
Figure 4.2: Cumulative distribution curves showing the number of impacts per year per square kilometer on an asteroid surface located in the Main Belt, for a given impactor size (or greater). In this work I use an impactor population produced by the modeling described in O’Brien and Greenberg (2005) (solid), which has a cumulative log-log slope of -2.93 for impactor diameters $D_p$ in the range $0.1 \text{ m} < D_p < 95 \text{ m}$, and shallowing to a cumulative log-log slope of $\sim -1$ for impactor diameters $D_p > 95 \text{ m}$. Also plotted are the similar impactor populations used by Belton et al. (1992) (dashed) and Greenberg et al. (1994, 1996) (dotted) to model the cratering records on the asteroids 951 Gaspra and 243 Ida.
Figure 4.3: A plot of the simple 30× scaling-law used in this study for mapping impactor size to crater size (solid), showing it in comparison to strength scaling (Holsapple, 1993), gravity scaling (Holsapple, 1993; Melosh, 1989), and the hydrocode simulations for 951 Gaspra (dot-dashed) from Greenberg et al. (1994) and 243 Ida (dashed) from Greenberg et al. (1996). The gray bands represent a variety of target material types, from loose sand to competent rock. Note that the size of impactors used in my stochastic cratering model range from 0.667 m (producing 20 m craters) to 667 m (producing 20 km craters).
layers, until superposing begins to occur. In that case, the program decides whether a new crater actually erases a portion of an older, pre-existing crater (1st generation) or is just superposed upon the surface of the older crater (2nd generation) with the older crater intact underneath. In order to erase a portion of a larger pre-existing crater, the new crater must have a diameter which is at least 1/10 the diameter of the pre-existing crater (Soderblom, 1970). If erasure or writing to a blank surface occurs, then the new crater is written onto both 1st and 2nd generation layers, while if a superposition only occurs, then the new crater is written only onto the 2nd generation layer. If a 2nd generation crater is erased by ejecta or seismic shaking later, it is ‘cleaned off’ the 2nd generation layer and any pre-existing 1st generation crater underneath is restored to the master 2nd generation layer above. All of these events occur on a element by element basis, such that once a crater is implanted, it is treated only in a unit area fashion from that moment on, not as an entire feature.

This erasure and superposition method is based upon the principle of impact gardening (see Melosh (1989) for details) and the crater erosion work described in Soderblom (1970), in which each new crater will only affect and turn over a depth of rock and regolith which is about 1/8-1/12 of its diameter. For example, while new, very small craters (tens of meters in diameters) can superpose themselves on the larger craters on Eros, such as Himeros (≈ 10 km diameter), Shoemaker (7.6 km diameter), and Psyche (5.3 km diameter), they cannot erase these features—it takes a new impact on the same size scale as the old one to actually erase a portion of it, such as the portion of Himeros which has been removed by the emplacement of Shoemaker.

4.1.5 Erasure by crater ejecta coverage

Whenever a new crater is formed (whether 1st generation or 2nd generation), the area within 5 crater radii of the impact site is affected by and examined for possible crater erasure by the ejecta blanket emplaced by the new impact. The ejecta blanket
thickness (height) as a function of distance from the new crater center is found using Eq. 6.3.2 from Melosh (1989):

\[ H_b = 0.14 R^{0.74} \left( \frac{r}{R} \right)^{-3}, \]  

\[ (4.2) \]

where \( H_b \) is the ejecta blanket thickness, \( r \) is the distance from the crater center, and \( R \) is the new crater radius. Note that this equation is based upon lunar craters, and will therefore require an upgrade as additional work is done on asteroid craters and ejecta (see Ch. 6). It also assumes gravity dominated cratering only—a problematic assumption on its own, especially near the small crater end of the asteroid cratering scale.

If I assume that a crater is parabolic in shape and has a \( d/D \) ratio of 1/5, then the crater volume \( V \) and area \( A \) are given by:

\[ V = \frac{1}{40} \pi D^3, \]

\[ (4.3) \]

\[ A = \frac{1}{4} \pi D^2. \]

\[ (4.4) \]

Dividing \( V \) by \( A \) implies that if the area of the crater \( A \) is covered by ejecta to a depth of 1/10 \( D \), the crater should be effectively filled (perhaps with agitation by seismic shaking from subsequent impacts). Two matrices (one for 1st generation and one for 2nd generation craters) keep track of the total amount of ejecta collected per unit surface area (crater element) in each crater, and when the depth in a given mesh element reaches 1/10 \( D \) of the crater in question, that unit area is considered filled or erased.’

This ejecta erasure method is admittedly rather simplistic, but adding to the sophistication of the method with our current knowledge level regarding ejecta blanketing on small bodies is unwarranted. As this gap is filled in, a more complicated method can be incorporated.
4.1.6 Erasure by seismic shakedown

Two separate matrix layers are used to track seismic damage to both 1st generation and 2nd generation craters. If a new crater’s impactor is at least 1 m in diameter, then all other craters on the 1st and 2nd generation surfaces receive seismic damage in accordance with Eq. 3.14; that is, a value of downslope diffusion $K_i$ is computed for each new impact, and this value is added to the total diffusion sum $K$ for all mesh elements outside of the new crater’s radius. If the total diffusion sum $K$ for a particular crater element equals or exceeds the value computed by the right side of Eq. 3.16 for that crater, then that unit area is considered filled or erased.

4.1.7 Observation rules

Because the craters produced by this model are handled in a per-unit-area (pixel) fashion during crater production and erasure, some rules need to be applied when they are counted for the production of a crater size-frequency distribution curve. Actual crater counting requires that at least enough of the rim of the crater remains, such that a radius of curvature can be estimated within the resolution limits of the image being examined. To incorporate a semblance of this feature of crater counting into this model, I use simple rules specifying that some fraction of the original crater area must remain in order to be counted. These rules are currently that >25% of craters >200 m in diameter must remain to be counted, >50% of craters between 130-200 m must remain to be counted, >75% of craters between 100-130 m must remain to be counted, and that 100% of craters <100 m must remain to be counted. Note that these rules are heavily affected by the pixelation of the craters, especially for craters less than 100 m (5 pixels across). These observation rules were derived practically by calibrating the model at empirical crater saturation levels (without seismic shaking) to produce a relatively straight log-log cumulative distribution at about 7-8% of geometric saturation (for discussion see (Melosh, 1989)).
4.1.8 Global cratering record resets?

At the large end of the impactor scale, the vertical velocities imparted to the rigid block in the Newmark slide-block model begin to approach the escape velocity of the asteroid. In these instances, it is assumed that the Newmark model begins to break down, and that a global upset and re-arrangement of the asteroid’s regolith (and some underlying fractured bedrock) will occur. This effect was estimated in Greenberg et al. (1994) and then used to explain the usual shape of the size-frequency distribution of craters on 951 Gaspra. This effect requires further investigation, and has not yet been implemented in the impact cratering model.

4.2 End member testing

Prior to using this cratering evolution model to look at the cratering record on the surface of Eros, I performed some general ‘end-member’ impactor population testing to verify that the cratering behavior in the model corresponds to past known experimental and numerical modeling results. The two ‘end-member’ impactor populations tested are shown in Fig. 4.4, one having a cumulative distribution slope of -3 and one having a cumulative distribution slope of -1. That is, I purposefully tested the effects on a cratering record of having impactor distributions which are steeper (in the case of -3) and less steep (in the case of -1) than the -2 cumulative distribution typical of both ‘geometric saturation’ and ‘empirical saturation’ (Melosh, 1989). The expected behavior in each of these cases is generally described in Chapter 10 of Melosh (1989) (along with the original references), providing me with a means for testing the effectiveness of my particular stochastic cratering model.

4.2.1 Steep slope impactor population test

Regardless of the distribution slope of the impactor population, early on in the cratering evolution of the body, the slope of the crater population will directly
Figure 4.4: A cumulative distribution plot of the two ‘end-member’ impactor popu-
lations used to test the stochastic cratering model used in this study, one having a
slope of -3 and one having a slope of -1. Note that the overall magnitude (vertical
position) of these populations are relatively close to the Main Belt population shown
in Fig. 4.2, and will therefore produce time scales for cratering which are of the same
order as the more complex impactor population used for simulating the statistics of
the cratering record on the surface of 433 Eros.
reflect, or mimic, the slope of the impactor population. This is called a crater ‘production population’ for the target body, and is illustrated in Fig. 4.5. Note that were it not for crater saturation, the crater population would continue to maintain this high -3 slope indefinitely, with ever increasing magnitude (crater numbers). However, relatively early on in the process, the number of small craters reaches what is generally referred to as ‘empirical saturation,’ that is, the observed number of craters in this size range reaches an equilibrium point (erasure rate = production rate). As time progresses, larger and larger crater diameters also reach empirical saturation, which has a slope of -2 (cumulative) and a magnitude generally between 5% and 10% of geometric saturation (Melosh, 1989). This behavior produces a ‘knee’ in the crater distribution, with small craters in empirical saturation (cumulative slope of -2) and larger craters not in empirical saturation (cumulative slope of -3). This behavior is illustrated in Fig. 4.6. Over the course of time, the ‘knee’ in the curve walks all the way out to the largest crater sizes on the asteroid, such that the cratering record on the asteroid is in a complete state of empirical saturation. This condition is shown in Fig. 4.7.

The primary reason for this type of empirical saturation behavior is the erasure of craters of one particular size by the superposition of (or degradation by) craters of a smaller size (See Sec. 4.1.4). This form of crater erosion is well understood and has been experimentally demonstrated in Gault (1970) and analytically demonstrated in Soderblom (1970). Where the empirical saturation level falls (what percentage of geometric saturation) is a function of the material being bombarded, the regolith thickness, and the observation resolution—how well can craters be seen. In Figs. 4.6 and 4.7 two lines are plotted for the crater distributions: the thin line show the actual distribution of craters down to the resolution of the model (20 m pixel-scale), while the thick line shows the observed crater distribution after a set of observation rules have been applied (see Sec. 4.1.7. At the resolution of the model, the empirical saturation level coincides with about 20-50% of geometric saturation;
Figure 4.5: Display screen from the stochastic cratering model showing (left) a cumulative distribution plot of the craters on the model surface and (right) a pictorial representation of the model surface, with craters color coded by size (using spectral colors with blue for small craters and red for large craters). The dashed line represents the geometric saturation line (maximum possible crater density), and the dotted lines represent typical values of ‘empirical saturation,’ which generally falls between 5% and 10% of geometric saturation. At this early time in the model run, the crater population distribution matches the impactor population in slope (-3 cumulative) and is called a production population.
Figure 4.6: Display screen from the stochastic cratering model showing (left) a cumulative distribution plot of the craters on the model surface and (right) a pictorial representation of the model surface, with craters color coded by size. The dashed line represents the geometric saturation line (maximum possible crater density), and the dotted lines represent typical values of ‘empirical saturation,’ which generally falls between 5% and 10% of geometric saturation. Two lines are plotted for the crater distributions: the solid thin line shows the actual distribution of craters down to the resolution of the model (20 m pixel-scale), while the thick line shows the observed crater distribution after a set of observation rules have been applied (see Sec. 4.1.7. At this stage in the model run, the crater population has reached empirical saturation at small sizes, while a production population still exists (with some stochastic variation) at larger sizes.
Figure 4.7: Display screen from the stochastic cratering model showing (left) a cumulative distribution plot of the craters on the model surface and (right) a pictorial representation of the model surface, with craters color coded by size. The dashed line represents the geometric saturation line (maximum possible crater density), and the dotted lines represent typical values of ‘empirical saturation,’ which generally falls between 5% and 10% of geometric saturation. Two lines are plotted for the crater distributions: the solid thin line shows the actual distribution of craters down to the resolution of the model (20 m pixel-scale), while the thick line shows the observed crater distribution after a set of observation rules have been applied (see Sec. 4.1.7. At this stage in the model run, the crater population has reached empirical saturation at all levels.
however, after the application of observation rules this level drops down to about 5-10% of geometric saturation, consistent with previous (primarily lunar) studies (Melosh, 1989).

An interesting feature of this model appears when a very large impact event occurs. In these instances, the large area of the new crater itself, and the large amount of ejecta produced by the impact produces a significant, temporary drop in the overall crater counts. The smallest craters are the first to begin recovering from this sudden blow to their numbers, followed by the larger crater sizes. This creates a concave-up crater distribution for craters sizes smaller than the new large crater (see Fig. 4.8). Within a relatively short amount of time (a few million years or so in this instance) the crater counts recover from this large impact and again resume their empirical saturation. This ‘punctuated equilibrium’ form of behavior was initially described by Schultz et al. (1977) for small craters on the lunar maria, and was proposed as a possible explanation for the steep crater distribution on Gaspra by Greenberg et al. (1994)—that is, the steep crater distribution on Gaspra is consistent with the asteroid having suffered a relatively recent, very large impact.

4.2.2 Low slope impactor population test

When the distribution slope of the impactor population is low (-1 slope cumulative), early on in the cratering evolution of the body the slope of the crater population will (again) directly reflect, or mimic, the slope of the impactor population. As in Sec. 4.2.1, this gives a crater ‘production population’ for the target body, and is illustrated in Fig. 4.9.

As this model progresses, the behavior of the crater population becomes vastly different from that shown in Sec. 4.2.1. The empirical saturation levels, which are produced by the erasure of larger craters by smaller craters, no longer apply, such that the cumulative level of small craters continuously varies, and is controlled primarily by the superposition of larger craters over areas of smaller craters (see
Figure 4.8: Display screen from the stochastic cratering model showing (left) a cumulative distribution plot of the craters on the model surface and (right) a pictorial representation of the model surface, with craters color coded by size. The dashed line represents the geometric saturation line (maximum possible crater density), and the dotted lines represent typical values of ‘empirical saturation,’ which generally falls between 5% and 10% of geometric saturation. Two lines are plotted for the crater distributions: the solid thin line shows the actual distribution of craters down to the resolution of the model (20 m pixel-scale), while the thick line shows the observed crater distribution after a set of observation rules have been applied (see Sec. 4.1.7. This figure shows the short-term effect of a very large impact on the overall cratering record, creating a temporary, concave-upward segment in the crater distribution for craters below the size of the large impact. Following this temporary dip, the levels quickly recover to their original empirical saturation values—beginning at the smallest sizes and working its way up in sizes. I have named this large-impact effect ‘punctuated equilibrium.’
Figure 4.9: Display screen from the stochastic cratering model showing (left) a cumulative distribution plot of the craters on the model surface and (right) a pictorial representation of the model surface, with craters color coded by size (using spectral colors with blue for small craters and red for large craters). The dashed line represents the geometric saturation line (maximum possible crater density), and the dotted lines represent typical values of ‘empirical saturation,’ which generally falls between 5% and 10% of geometric saturation. Two lines are plotted for the crater distributions: the solid thin line shows the actual distribution of craters down to the resolution of the model (20 m pixel-scale), while the thick line shows the observed crater distribution after a set of observation rules have been applied (see Sec. 4.1.7. At this early time in the model run, the crater population distribution matches the impactor population in slope (-1 cumulative) and is called a production population. Note the significant difference between this figure and Fig. 4.5.
This behavior can be described as a form of ‘dynamic equilibrium,’ in which the small crater numbers vary significantly about a relatively flat, low level. At the same time, the largest craters (which are not over-written by even larger ones) approach geometric saturation values on the surface (also shown in Fig. 4.10).

This type of rather unusual, cratering statistics behavior was described in a limited fashion in Chapman and McKinnon (1986), and further expanded upon in Chapter 10 of Melosh (1989). While it is immediately apparent that this form of cratering behavior has /textit{not} been observed cratered bodies that have been imaged thus far (none have this type of crater distribution), there is some evidence that suggests that the impactor population in the asteroid belt does flatten out at larger sizes (greater than 100 meters in diameter) – see Sec. 4.1.2. There is also some supporting evidence for this idea shown in the cratering record on the asteroid 253 Mathilde, which has an as-yet, unexplained ability to survive very large impacts, and which has large crater levels which are very near to geometric saturation levels (similar to the large crater values shown in Fig 4.10).
Figure 4.10: Display screen from the stochastic cratering model showing (left) a cumulative distribution plot of the craters on the model surface and (right) a pictorial representation of the model surface, with craters color coded by size. Two lines are plotted for the crater distributions: the solid thin line shows the actual distribution of craters down to the resolution of the model (20 m pixel-scale), while the thick line shows the observed crater distribution after a set of observation rules have been applied (see Sec. 4.1.7. The dashed line represents the geometric saturation line (maximum possible crater density), and the dotted lines represent typical values of ‘empirical saturation,’ which generally falls between 5% and 10% of geometric saturation. At this late time in the model run, the crater population distribution displays a form of ‘dynamic equilibrium,’ in which the large crater values approach geometric saturation, while the small crater values are continually being ‘knocked back down’ by the superposition of very large craters.
CHAPTER 5

FINAL STUDY RESULTS

5.1 Understanding the Eros cratering record

Equipped with a workable model for the evolution of craters on the surface of an asteroid (Chapter 4), the next phase of the modeling involved an attempt to recreate the statistics of the cratering record on the surface of 433 Eros, included the observed paucity of small craters. Example screens from the stochastic cratering model are shown in Figs. 5.1 and 5.2, which display model runs without and with (respectively) the effects of seismic shakedown included. A relative size-frequency distribution plot (R-plot) of the model output is also shown in Fig. 5.3 for the case which includes the effects of seismic shakedown (Arvidson et al., 1979). In both figures, the observed (Chapman et al., 2002; Robinson et al., 2002) and modeled distributions are in good agreement (for craters > 100 m) at a Main Belt exposure age of 400 ± 200 Myr. Note that this age is neither an absolute age for Eros, since major impacts can potentially ‘reset’ the asteroid surface record (Greenberg et al., 1994, 1996), nor is it an accurate age for the current surface record, because the chaotic nature of Eros’s orbit has exposed it to a highly variable impactor flux (Michel et al., 1998)—rather than the assumed constant flux used in my modeling. At best, this surface age represents a lower limit, assuming that an average Main Belt impactor flux is at the high end of what Eros has actually been exposed to. The observed and modeled crater populations differ at larger crater sizes (> 2 km), where the statistics of small numbers and random crater generation make matching the observed crater population at this end of the curve difficult. The simulations lead to two possibilities to explain this mismatch:
1. the age obtained could be an absolute Main Belt exposure age for Eros, and
   I have simply not had a model run which generates the correct large crater
   population to match the observed distribution, or

2. the number of very large craters on Eros is indicative of a much older body
   than the current cratering record reflects, such that the measured Main Belt
   exposure age reflects the time elapsed since the last major impact event, which
   'reset' the small crater record (Greenberg et al., 1994, 1996).

Additional modeling tends to support the latter interpretation, in that good
matches to the large crater record (> 2 km) are obtained after about 1-2 Gyr. At
these model ages, however, the 'dip' in the Eros cratering record between 1 and 6
km diameters is replaced by standard 'empirical saturation' levels, since the current
model does not simulate global cratering record resets from very large impact events.

Note that it is only in Fig. 5.2 and Fig. 5.3–where the effects of seismic shakedown
are included–that the observed small cratering record is matched by the models. The
reduced number of small craters is a result of seismic erasure (cumulative seismic
damage via Eqs. 3.14 and 3.16), causing lower equilibrium crater numbers for craters
< 100 m diameter than would otherwise be expected (empirical saturation). This
equilibrium point is a sensitive function of the assumed thickness of the mobilized
regolith layer $h$. By varying this parameter, I find a best fit corresponding to
$h \approx 0.1 \text{ m}$, with actual values for $h$ perhaps as high as a few meters (due to the
assumptions and uncertainties in our modeling). This thickness is significantly less
than the (somewhat rough) estimates of an average regolith thickness of 20-40 m
from the NEAR observations (Thomas et al., 2002; Robinson et al., 2002). I infer
from this difference that much of the regolith layer possesses a depth-dependent
porosity and cohesion gradient, perhaps due to compaction from seismic shaking.
This characteristic would produce lower porosity and higher cohesion with increasing
depth (Robinson et al., 2002). Such a gradient was observed for the lunar regolith
Figure 5.1: Example screen from the stochastic cratering model following 400 Myr of impacts on an Eros-like target body. The area shown in these displays is 34 km × 34 km (1156 km²), with a model resolution of 20 m × 20 m. Craters are color coded by size (see color bar). The curves shown in the cumulative distribution plots are: geometric saturation (dashed), estimates of empirical saturation at 5% to 10% geometric saturation (dotted), modeled crater distribution (thick solid), and the observed crater distribution on 433 Eros (dot-dashed) from Chapman et al. (2002); Robinson et al. (2002). This figure shows the crater population after 400 Myr without seismic shakedown included in the simulation. Note the empirical saturation level of small craters and a poor match with the observed distribution of craters on Eros.
Figure 5.2: Example screen from the stochastic cratering model following 400 Myr of impacts on an Eros-like target body. The area shown in these displays is 34 km × 34 km (1156 km²), with a model resolution of 20 m × 20 m. Craters are color coded by size (see color bar). The curves shown in the cumulative distribution plots are: geometric saturation (dashed), estimates of empirical saturation at 5% to 10% geometric saturation (dotted), modeled crater distribution (thick solid), and the observed crater distribution on 433 Eros (dot-dashed) from Chapman et al. (2002) and Robinson et al. (2002). This figure shows the crater population after 400 Myr with seismic shakedown included in the simulation. Note the excellent match between the modeled and observed crater distributions, particular with regard to small craters (less than 100 m diameters).
Figure 5.3: A relative size-frequency distribution plot (Arvidson et al., 1979) of 433 Eros craters per square kilometer as a function of crater diameter, showing a favorable comparison between observed (Chapman et al., 2002; Robinson et al., 2002) and modeled values after 400 ± 200 Myr (Main Belt exposure age). The low abundance of small craters is a result of seismic erasure, causing lower equilibrium values than would otherwise be expected (empirical saturation; thin dashed line). Symbols are the same as those listed in Table 1 of Chapman et al. (2002). Based on Fig. 4 of Richardson et al. (2004).
(down to sampling depths of a few meters), causing the regolith to preferentially slide at shallow 'critical' depths (Houston et al., 1973).

It should be pointed out that based upon this study, large asteroids greater than about 70-100 km diameter (see Figs. 2.6, 3.4, 3.5) should not experience the same degree of small crater erasure as asteroids in the size range of Eros, and should therefore have small crater equilibrium numbers approaching more typical 'empirical saturation' levels (the dotted lines in Fig. 5.1. Such bodies should only experience local seismic effects from all but the largest impacts, and not the global seismic effects modeled in this study. As an example of a much larger body, the Earth's moon does show some minor, local variations in small crater numbers depending upon the surface material and age; however, most areas display rather typical levels of empirical saturation at small crater diameters (5-200 m) when observation effects, such as sun angle, are taken into account (Schultz et al., 1977).

5.2 The enigma of ponded deposits on Eros

One of the most extraordinary finds of the NEAR-Shoemaker mission was the ponded deposits found in small craters up to a few hundred meters in diameter (see Figs. 5.4, 5.5). These ponds show clear evidence of downslope material motion, and impact-induced seismic shaking has been suggested as a possible means for their formation (Cheng et al., 2002b). However, the ponds display several features, described in Robinson et al. (2001), that differ from the model of downslope motion and crater erosion that I have established in this work:

- a difference in geomorphology: the ponds appear as extremely flat deposits in the bottoms of craters and topographic lows, while craters degraded by seismic shakedown maintain their curving, bowl shape and slowly 'soften' over time (Figs. 3.9, 3.10).

- an apparent difference in flow style: the very flat ponded deposits are indicative...
of a frictionless, fluid-like flow, while the downslope flow induced by seismic shaking involves frictional forces and requires a slope (or gradient) to occur (Sec. 3.3).

- a difference in localities: the ponded deposits are concentrated primarily in areas of prolonged terminator exposure and low surface gravity, while impact-induced seismic effects should be global in distribution.

- a difference in grain-size: the ponds appear visually to be composed of finer-grained material than their surroundings, a conclusion supported by their bluer color, while debris flow induced by seismic shaking would tend to be more assorted (or even cause the larger grain sizes to rise to the top via the ‘Brazil Nut’ effect (Asphaug et al., 2001)).

These features all lend support to electrostatic dust levitation (Lee, 1996) as the mechanism for the formation of ponded deposits on Eros, as suggested in Robinson et al. (2001). There may be areas where the two effects are mixed, with seismic shaking acting as the bulk mover of material downslope and the primary agent of crater erosion, but with electrostatic dust levitation producing a veneer of thin, flat deposits during the time periods between major impacts. Another possibility is that while Eros was located in the Main Asteroid Belt, seismic shaking was the dominant force for downslope flow and surface modification. However, now that Eros has moved to the Near Earth Asteroid environment, the impactor flux has been reduced by a factor of $10^2$ to $10^3$ (Ivanov et al., 2002) and the solar flux has increased by factor of about four. Thus electrostatic dust levitation could currently be the dominant surface modification mechanism on the asteroid. At the least, the much longer (on average) intervening time between major impacts permit electrostatically formed ponds to form and grow to larger sizes than previously, without seismic disruption.
Figure 5.4: An example of a ponded deposit on 433 Eros, imaged by the NEAR-Shoemaker spacecraft. Note the marked difference in morphology between this pond and the degraded craters shown Figs. 1.3, 1.4, 3.9, and 3.10 indicative of different formation processes. This image shows a beautiful 100 m diameter ponded deposit containing an embedded 25 m boulder. Note the extremely flat surface containing a tiny (few-meter diameter) impact crater (MET 155888598, 179.04 W, 2.42 S, 0.55 m/pixel).
Figure 5.5: An example of a ponded deposit on 433 Eros, imaged by the NEAR-Shoemaker spacecraft. Note the marked difference in morphology between this pond and the degraded craters shown Figs. 1.3, 1.4, 3.9, and 3.10 indicative of different formation processes. This image shows a small, 75 m diameter ponded deposit. Note the difference between the smooth, fine-grained pond surface and the coarse, boulder strewn terrain surrounding the deposit (MET 155888731, 183.88 W, 3.21 S, 0.63 m/pixel).
5.3 Concluding remarks on seismic shakedown

For 433 Eros, the modeling work presented here has produced excellent agreement with the empirical observations, particularly with regard to the time evolution of crater morphology and the statistics of the impact cratering record. Nevertheless, I note that there is considerable uncertainty with regard to the asteroid’s actual seismic and regolith properties. I have based my results on values appropriate to the one impact-generated environment that has been studied in detail: the upper lunar crust. Even with these uncertainties, however, this work constrains these properties and effectively demonstrates the ability of impact-induced seismic shaking of an Eros-sized asteroid to destabilize slopes, cause regolith to migrate downslope, and to degrade and eventually erase small impact craters on the surface. Additionally, this result lends support to an internal ‘fractured monolith’ structure for Eros, and also supports asteroid population models that produce a steep population of small asteroids despite removal by the Yarkovsky effect.

While the true efficacy of this mechanism (and the accuracy of my modeling) will not be shown until seismometers are landed on asteroid surfaces and their response to artificial and natural impacts measured, an earlier test of this work will occur when detailed images of large Main Belt asteroids (greater than 70-100 km diameter) are returned: such asteroids should exhibit less small-crater erasure, due to the lack of global seismic effects, and should therefore have small crater numbers more indicative of the actual small impactor population (or show more typical empirical saturation levels).

This modeling also demonstrates the potential of seismic studies of asteroids to investigate their interiors. Such studies could not only give us information about the internal structure of these bodies; such as major fracture boundaries, internal stratigraphy, voids, and small-scale fracture spacing; but could also provide information about the rock composition, compositional variations, elastic response, and
seismic dissipation properties. Both reflection and standard seismological techniques could be employed, building upon our experience with the Apollo seismic experiments and taking advantage of the advances that have occurred in the field since that time. I look forward to the openings that this tool offers toward our understanding of these fascinating objects.
CHAPTER 6

IMPACT EJECTA BEHAVIOR MODELING FOR THE DEEP IMPACT MISSION

6.1 Introduction

In addition to investigating the seismic behavior of asteroids in response to impacts, and their effects on asteroid surface morphology, I have also been performing numerical modeling for the Deep Impact mission to Comet 9P/Tempel 1. The goal of this mission is to purposefully collide a small ‘impactor’ spacecraft with the comet while simultaneous observing the event from a ‘flyby’ spacecraft—in effect, we hope to perform a large, in-situ impact experiment on a cometary body. One important aspect of this mission is analyzing the behavior of the impact ejecta produced by the crater excavation process. Following impact, individual ejecta particles are launched ballistically from the edge of the bowl of the expanding crater, and collectively these particles form an inverted, cone-shaped plume (or curtain) which also expands over time (Fig. 6.1). The ballistic behavior of the individual particles and the collective behavior of the ejecta plume are both heavily affected by the velocity and characteristics of the impactor (which we control); and target parameters such as the density, strength, porosity, and gravity field. By modeling the observed behavior of individual ejecta particles and the collective ejecta plume resulting from the mission, I hope to place constraints on these parameters.

6.2 Impact ejecta scaling-laws

To model the impact ejecta behavior, I develop a revised set of crater growth, ejection time, and ejecta velocity scaling-laws, based upon the scaling-laws described
Figure 6.1: The debris ejected from an impact crater follows ballistic trajectories from its launch position within the transient crater (horizontal and vertical scales are in units of \( R \)). The innermost ejecta are launched first and travel fastest, following the steepest trajectories shown in the figure. Ejecta originating farther from the center are launched later and move more slowly, and fall nearer to the crater rim. Because of the relationship between the position, time, and velocity of ejection, the debris forms an inverted cone that sweeps outward across the target. This debris curtain (plume) is shown at four separate times during its flight, at 1, 1.5, 2, and 2.5 \( t_f \), where \( t_f \) is the crater formation time. Figure reproduced from (Melosh, 1989).
in Housen et al. (1983). The two relationships used here are based upon gravity-dominated cratering in an experimental environment and are given by:

Crater formation time:

$$ t_f = \frac{1}{2} \sqrt{\frac{2R}{g}} , $$

(6.1)

Ejecta velocity as a function of crater radius:

$$ v_{ej} = C_c \left( \frac{r}{R} \right)^{-\epsilon} $$

(6.2)

where $R$ is the gravity-dominated transient crater radius ($\frac{D}{2}$), $r$ is the crater radius as a function of time, $g$ is the surface gravitational acceleration, and $\epsilon$ is a material constant ranging from 1.8 for competent rock to 2.6 for quartz sand (Melosh, 1989). The coefficient of $\frac{1}{2}$ in front of the crater formation time equation Eq. 6.1 comes from an empirically derived value of 0.54 given in Melosh (1989).

I find the constant $C_c$ by assuming that the crater rim advancement velocity must be equal to the horizontal component of the particle ejection velocity, such that the ejecta plume base and crater rim advance at the same rate. I take advantage of this by setting the particle ejection angle to a mean of $\theta = 45^\circ$ above the horizon, which gives $v_{\text{horizontal}} = \frac{\sqrt{2}}{2} v_{ej}$. Letting $v_{\text{horizontal}}$ equal the rim advancement speed produces:

$$ \frac{\delta r}{\delta t} = \frac{\sqrt{2}}{2} C_c \left( \frac{r}{R} \right)^{-\epsilon} . $$

(6.3)

Solving this differential equation such that $r$ is allowed to move from 0-$r$ while $t$ moves from 0-$t$, yields:

$$ r = \left[ \frac{\sqrt{2}}{2} (1 + \epsilon) C_c R^{\epsilon} t \right]^{1/\epsilon} . $$

(6.4)

Letting $r = R$ and $t = t_f$ produces a crater formation time of:

$$ t_f = \frac{\sqrt{2}R}{C_c(1 + \epsilon)} . $$

(6.5)
However, $t_f$ is also given by Eq. 6.1, allowing a solution for $C_e$ to be found:

$$C_e = \frac{2\sqrt{Rg}}{1 + \epsilon}. \quad (6.6)$$

This gives the following two model equations, in addition to Eq. 6.1:

**Ejecta velocity as function of crater radius (replaces Eq. 6.7)**

$$v_{ej} = \frac{2\sqrt{Rg}}{1 + \epsilon} \left(\frac{r}{R}\right)^{-\epsilon}, \quad (6.7)$$

**Particle ejection time as a function of rim position:**

$$t_{ej} = \frac{\sqrt{2^{r+1+\epsilon}}}{2\sqrt{gR^{\epsilon+\frac{3}{2}}}}. \quad (6.8)$$

I further modify Eq. 6.7 to simulate late-stage ejection velocities more properly, when the crater radius is approaching its final value. In gravity-dominated cratering, the particle ejection (ballistic) velocity should go to zero as $r$ goes to $R$, while Eq. 6.7 instead goes to a constant (a weakness also described in Housen et al. (1983)). I correct this by subtracting a higher order term, which has negligible effect throughout most of the excavation process, but which ramps the velocity expression to zero as the transient crater rim (at $R$) is approached—essentially applying a mathematical bridge between known good behaviors. This gives (replacing Eq. 6.2 and Eq. 6.7):

$$v_{ej} = \frac{2\sqrt{Rg}}{1 + \epsilon} \left(\frac{r}{R}\right)^{-\epsilon} - \frac{2\sqrt{Rg}}{1 + \epsilon} \left(\frac{r}{R}\right)^{\lambda}, \quad (6.9)$$

where the power $\lambda$ is selected to produce smooth ejection velocity behavior (down to zero velocity) over the final stages of crater formation, as $r$ approaches $R$ (typical value: $\lambda \approx 6$-$10$). Fig. 6.2 shows a plot of ejection velocities produced from Eq. 6.9, compared to experimentally derived values.

### 6.3 Ejecta plume tracer model

These scaling-law equations are then applied to a dynamical simulation which models—via thousands of point tracer particles—the ejecta plume behavior, ejecta
Figure 6.2: (top) Normalized (non-dimensional) ejecta velocities produced using Eq. 6.9 as a function of the normalized radial position r within the transient crater of radius R. The values computed are for loose sand, with ε = 2.44. (Bottom). Experimentally measured ejecta velocities produced from large explosion craters and published in Fig. 4 of Housen et al. (1983). The best model-experiment agreement corresponds to the lower sand target values.
blanket placement, and impact crater area resulting from a specified impact on an irregularly shaped target body (similar to Geissler et al. (1996)). Fig. 6.3 shows an example of one impact simulation, visualized in three-dimensional polygon fashion. Placing the target body (shape-model) into a simple rotation state about one of its principal axes, the user then inputs an impact site and a set of projectile/target parameters. From this information, the program places a circular transient crater area on the surface and populates this area with random tracer particles that have a spatial distribution such that each particle represents a roughly equal volume of ejecta. Once positioned, each particle is assigned an ejection time (Eq. 6.8, velocity (Eq. 6.9), and direction (radially outward at ejection angle 30°-60° above the horizon: discussed below), after which the simulation clock begins. While in flight, the gravitational acceleration from the irregular target body on each tracer particle is computed using the polygonized surface (polyhedron) gravity technique developed by Werner (1994). The model tracks all tracer particles until they have either left the gravitational sphere of influence of the body (escaped) or landed again on the surface.

To properly model the ejection angle variations that occur over time in impact cratering experiments (Cintala et al., 1999; Anderson et al., 2003), I mimic the empirical data by allowing the particle ejection angle to drop from \( \theta = 60^\circ \) to \( \theta = 30^\circ \) as the crater rim \((r)\) moves from 1 projectile diameter (its starting point) to the transient crater radius \((R)\). This feature also causes the ejecta plume shape to change as a function of time, demonstrated in the Cintala et al. (1999) experiments and compared to the model in Fig. 6.4. If a gravity-dominated cratering event occurs as a result of our impact on Tempel 1, the shape of the ejecta plume will provide a means for marking the end of the crater formation process (end of excavation flow) as the ejecta plume changes shape from concave during excavation, to straight at the transient crater rim (end of excavation), to convex during the post excavation (fall-out) stage.
Figure 6.3: A simulation showing the ejecta plume (white tracer particles), ejecta blanket (yellow tracer particles), and impact crater surface area (shown in blue) resulting from a small impact on an Eros-shaped target body having a 6-hour rotation period about its principal z-axis. The top panel shows the state of the ejecta 6 minutes after the impact, using 2000 tracer particles to map its behavior. The ejecta plume is fully formed at this stage, with the slowest particles beginning to fall out near the crater rim. The bottom panel shows the state of the ejecta 6 hours (one rotation) after the impact, with most of the tracer particles landed again on the surface to form the ejecta blanket. This blanket is slightly asymmetric, with more ejecta in the trailing direction (to the right) than in the leading direction.
Figure 6.4: Ejecta plume profile comparisons between the ejecta plume model (A) and a small-scale cratering experiment (B) performed by Cintala et al. (1999) and shown in their Fig. 11, in which a 3.18-mm glass sphere was shot into fine-grained sand at 1.24 m s$^{-1}$ to form an $\approx 4$ cm diameter crater. The top panel shows both horizontal and vertical scales normalized to the transient crater radius $R$, while the bottom panel shows a large vertical arrow at the transient crater rim (small arrows point out the ballistic paths of three individual ejecta particles)--with both figures roughly scaled to each other. The plume profiles in the top panel are shown in 4 ms increments, with the transient crater formed at about 44 ms (11 time steps). The plume profiles in the bottom panel are shown in 2 ms increments, with the transient crater formed at 45 ms (23 time steps). Note the change in plume profile from concave to convex as the transient crater rim is passed, along with a noticeable change in velocity with position—rapidly slowing as the transient crater rim is approached and gaining speed again as the slower particles fall out first. Compare these figures to the more basic Fig. 6.1.
In the event that the crater excavation is dominated by strength, I have added a target strength parameter \( R_s \) to the model, which cuts off crater growth and excavation flow when the inertial stress on the material reaches a user-assigned material yield stress \( Y \). This is determined by the equation (Melosh, 1989):

\[
R_s = \frac{\rho_i v_{ej}^2}{Y},
\]

where \( \rho_i \) is the surface density.

Unlike gravity-dominated excavation, in which the ejecta plume remains attached to the target surface throughout its formation and fall-out stages, in strength-dominated excavation the ejecta plume detaches completely from the target. In this case, the plumes bottom edge will follow a radial (with respect to the crater) ballistic path away from the edge of the truncated impact crater. While this form of cratering will give us a smaller final impact crater (and perhaps less chance of looking inside of the cavity), the ballistic path followed by the bottom edge of the ejecta plume (and perhaps some large late-ejected fragments) may provide us with our best opportunity for the determining magnitude of the comets gravity field.

Collectively, the expansion rate of the ejecta plume (especially in a gravity-dominated event) can itself be used to gain a measure of the surface gravity field \( g \), in part due to a \( g \) dependence in the particle ejection velocities (Eq. 6.9), but primarily due to the effect of gravity on the ballistic paths followed by the individual particles. Fig. 6.5 shows the effect of varying the gravitational force (by varying the density of a constant volume model) on the ejecta plume base position and velocity as a function of time. If this form of plume behavior can be observed at high enough resolution for several minutes following the impact, then a surface gravity and comet mass can be estimated (albeit roughly), as well as an approximation for the comets density by using the volume obtained from shape-modeling (Thomas et al., 2002).
Figure 6.5: Plots of model ejecta-plume position as a function of time (top) and ejecta-plume velocity as a function of time (bottom) for three different assigned densities to a homogeneous, ellipsoidal comet shape-model: 1.0 g cm$^{-3}$ (red), 1.5 g cm$^{-3}$ (green), and 2.0 g cm$^{-3}$ (blue). Dotted portions of the lines in the upper figure indicate the excavation (plume formation) stage, while the solid portions show the plume fall-out stage. The vertical black line at 800 seconds indicates the limit of observation time for the Deep Impact mission. Note the different expansion rates as a function of comet gravity, particularly evident in the velocity curves. The velocity curves also display a noticeable decrease during crater formation, an inflection point at the crater formation time $t_f$, and increasing velocity as slower particles fall out. With good ejecta plume resolution, I plan to use this type of plume behavior to estimate the gravity and mass of Comet 9P/Tempel 1.
6.4 Ejecta plume polygon model

A more realistic method for simulating the physical properties of an ejecta plume and eventual blanket resulting from an impact on a small, irregular target body is to model the ejecta plume as a three-dimensional polygon object, rather than randomly generated tracer particles. At each time step, the surface area and opacity of each polygon of the ejecta plume is calculated and rendered appropriately (assuming a user specified particle distribution). Fig. 6.6 shows an example of this model variant, for both a gravity-dominated and strength-dominated cratering event.

The polygon ejecta plume is initially formed by placing a mesh of 1800 regularly spaced tracer particles on the starting surface area of the crater, such that 3540 roughly equal-area triangular polygons are formed (each formed by connecting three tracer particles). This creates 59 rings of 60 polygons each, ranging from $\sqrt{\frac{1}{60}R^2}$ to R in radius-the hole in the center is intentional, in order to avoid the region of very fast ejecta particles which produce extremely deformed polygons and do not contribute significantly to the visible ejecta plume.

I currently calculate the mass of impact ejecta that each polygon represents by dividing the excavated portion of the crater into a series of simple paraboloid shells. The mass of material injected into each ring of 60 polygons is given by:

$$m_i = \frac{\rho \nu \pi}{8} (r_{i+1}^3 - r_i^3),$$  \hspace{1cm} (6.11)

where $m_i$ is the mass injected in ring $i$.

The mass per polygon in this ring is thus $m_o = (1/60) m_i$. This estimate is based upon the assumption that the excavation depth of the transient crater is about $D/8$ or $R/4$ Melosh (1989). The initial mass-loading per polygon will remain constant throughout the simulation, while the surface area of the polygon, $A$, changes dramatically throughout ejection and flight. Along with the target surface density $\rho_i$ (which is also used as a particle density), the model requires a particle mass distribution description. This is done by providing values for the smallest and largest
Figure 6.6: A simulation showing the ejecta plume as a 3-dimensional shape-model for a small gravity-dominated cratering event (top panel) and a small strength-dominated cratering event (bottom panel) on an Eros-shaped target body. An ejecta particle-size distribution has been assumed (maximum particle-size, minimum particle-size, and power-law distribution) with the resulting ejecta plume opacity calculated and rendered. Note that the ejecta plume detaches from the target body in the case of (bottom panel) strength-dominated cratering, with the first particles landing on the surface again at some distance from the impact crater site (if they do at all). In both simulations, a few random ejecta blocks are also included as discrete points. In the (top panel) gravity-dominated event, the convex shape of the ejecta plume indicates that the transient crater has finished forming and that the ejecta plume is now in the fall-out stage.
particle diameters in the plume (which are converted to minimum and maximum particle masses) along with a cumulative distribution power-law exponent. This is described by:

\[ dN = K m_p^{-b} \, dm_p \quad \text{(6.12)} \]

where \( N \) is the cumulative number of particles, \( K \) is a constant, \( m_p \) is the particle mass, and \( b \) is the supplied power-law exponent.

This gives us a means to derive the optical scattering properties of each ejecta plume polygon, which is a function of the scattering properties of each individual particles surface area and albedo. The surface area of an individual particle is given by:

\[ a = \pi \left( \frac{3 m_p}{4 \pi \rho_t} \right)^{\frac{2}{3}} \quad \text{(6.13)} \]

The total surface area per unit volume \( \sigma_v \) is found by solving \( d\sigma_v = a \, dN \), which yields:

\[ \sigma_v = \pi \left( \frac{3}{4 \pi \rho_t} \right)^{\frac{2}{3}} \left( \frac{K}{\frac{5}{3} - b} \right) \left( m_l^{\frac{5}{3} - b} - m_s^{\frac{5}{3} - b} \right) \quad \text{(6.14)} \]

where \( m_l \) and \( m_s \) are the mass of the largest and smallest particles respectively.

I solve for the constant \( K \) by placing it in terms of the mass density within the plume \( \rho_e \), using the relationship \( d\rho_e = m_p \, dN \):

\[ \rho_e = \left( \frac{K}{2 - b} \right) \left( m_l^{2-b} - m_s^{2-b} \right) \quad \text{(6.15)} \]

Solving this expression for \( K \), substituting back into the expression for \( \sigma_v \), and re-arranging to find the surface area per unit mass \( \sigma_m \) gives:

\[ \sigma_m = \frac{\sigma_v}{\rho_e} = \pi \left[ \frac{3}{4 \pi \rho_t} \right]^{\frac{2}{3}} \left[ \frac{2 - b}{\frac{5}{3} - b} \right] \left[ \frac{m_l^{\frac{5}{3} - b} - m_s^{\frac{5}{3} - b}}{m_l^{2-b} - m_s^{2-b}} \right] \quad \text{(6.16)} \]

which is an intrinsic property of the ejecta plume, based upon the user-supplied mass distribution of particles.
To determine the opacity of individual ejecta-plume polygons as a function of their changing surface areas $A$, I make use of the Lambert Exponential Absorption Law (Chamberlain and Hunten, 1987):

$$I_f = I_o e^{-\sigma_m \psi},$$  
(6.17)

where $I_o$ is the initial light intensity, $I_f$ is the final light intensity, and $\psi$ is the mass loading per unit area within the plume polygon \( \psi = \frac{m}{A} \). Note that $\sigma_m \psi$ is equivalent to the optical depth of the plume.

Normalizing the light intensity and bringing in the change in polygon area over the course of the simulation gives an opacity $O$ equation for each polygon:

$$O = 1 - e^{-\sigma_m \psi_o \left( \frac{A_o}{A} \right)},$$  
(6.18)

where $\psi_o$ and $A_o$ are the initial mass per unit area and initial polygon area respectively. Note that this opacity applies to viewing the plume surface from a normal (perpendicular) direction, and does not yet take into account the variable albedo of the particles to different light wavelengths. These inputs are supplied to a rendering tool (the OpenGL package), for visualization.

For my current modeling purposes, I use mass distribution values from a typical comet dust environment (Lisse et al., 2004) for the ejecta plume, although I expect that the actual observed plume will have a coarser particle distribution (and be correspondingly less opaque). When the actual observations are made, parameter searches using this forward model will be performed to better constrain these ejecta plume properties.

6.5 Instrument image sequence simulations

I also used this final form of the ejecta plume model in the planning of the instrument image sequences for the comet flyby spacecraft. This is done by modeling an impact on a shape-model target body and viewing it through a specifically designed display
module that simulates the flight path of the comet and flyby spacecraft, comet orientation and illumination, spacecraft orientation, and instrument field-of-view and mode information (Figs. 6.7, 6.8, 6.9, and 6.10). This sequence depicts one of our best possible scenarios: the comet presenting a large face-on profile, an excellent hit by the impactor, a well behaved gravity-dominated cratering event, very little image smear, and excellent impact site tracking for instrument pointing. What we actually see will most likely not be this ideal! Such image sequence modeling can also be used after the actual encounter to forward model many of the observed features from the impact and flyby.

6.6 Concluding remarks on ejecta plume modeling

In this chapter, I have described how impact ejecta scaling-laws and numerical modeling for the Deep Impact mission will provide us with a means to predict and analyze the observed behavior of the material launched from Comet 9P/Tempel 1 during crater excavation, and may provide us with a unique means of estimating the magnitude of the comets gravity field and by extension the mass and density of the comet. This work will also have direct application to impacts on asteroids, and will be used in future work to refine the cratering history modeling performed thus far. Recall that the impact cratering statistical model described in Ch. 4, Sec. 4.1.5 uses a very simple ejecta placement rule, as a function of distance from the new crater’s rim, which was based upon lunar crater data (Melosh, 1989). This rule is in obvious need of updating with regard to cratering on asteroids, particularly considering that cratering on asteroids does not proceed in a gravity-dominated fashion as it does on the lunar surface (see Sec. 4.1.3). The demonstrated competition between strength- and gravity-dominated excavation processes in the asteroid environment (Fig. 4.3), as explored in the hydrocode simulations for 951 Gaspra (Greenberg et al., 1994) and 243 Ida (Greenberg et al., 1996), implies that the ejecta plumes produced by craters on asteroids will be of the ‘detached’ type (shown in the bottom panel of
Figure 6.7: A sample image from an instrument sequence simulation for the Deep Impact mission High Resolution Instrument (HRI), showing false color renderings of a modified-Borelly comet shape-model (gray), transient crater surface area (black), ejecta plume (white), and several hundred random ejecta fragments (white). This first image shows the view about 30 seconds after impact, showing the forming crater and ejecta plume.
Figure 6.8: A sample image from an instrument sequence simulation for the Deep Impact mission High Resolution Instrument (HRI), showing false color renderings of a modified-Borelly comet shape-model (gray), transient crater surface area (black), ejecta plume (white), and several hundred random ejecta fragments (white). This second image shows the view shortly after transient crater formation is complete.
Pitch: 11.796 deg
Yaw: 0.017 deg

Figure 6.9: A sample image from an instrument sequence simulation for the Deep Impact mission High Resolution Instrument (HRI), showing false color renderings of a modified-Borelly comet shape-model (gray), transient crater surface area (black), ejecta plume (white), and several hundred random ejecta fragments (white). This third image shows what some of our best views of the interior of the crater might look like as the point of closest approach is rapidly passed.
Figure 6.10: A sample image from an instrument sequence simulation for the Deep Impact mission High Resolution Instrument (HRI), showing false color renderings of a modified-Borely comet shape-model (gray), transient crater surface area (black), ejecta plume (white), and several hundred random ejecta fragments (white). The fourth image shows the last possible image of the comet and ejecta plume as seen by the Medium Resolution Instrument (MRI) just before putting the spacecraft in Safe Mode at closest approach.
Fig. 6.6) more typical of strength-dominated cratering processes. My future work in this area will involve the use of the ejecta plume model described in this chapter to quantify and more fully describe the placement of ejecta resulting from impacts on asteroids and apply it to the stochastic model described in Chapter 4.
APPENDIX A

NEWMARK SLIDE-BLOCK MODEL CODE

The purpose of this code section is to compute the motion of a mobilized regolith layer (slide-block) resting on a slope which is seismically shaken per a given synthetic seismogram. The accelerations imparted to the layer (either through the local gravity or seismic shaking) are double integrated to compute the resulting displacements. Displacements occur in both the horizontal direction (in the reference frame of the slope), resulting in sliding; and the vertical direction, resulting in ballistic launching. The variables used in the code are listed below:

Asteroid surface gravity variables:
gh = horizontal gravity component (in the slope reference frame)
gv = vertical gravity component (in the slope reference frame)

Seismically shaken slope variables
ph = horizontal slope position
pv = vertical slope position
vh = horizontal slope velocity
vv = vertical slope velocity
ah = horizontal slope acceleration
av = vertical slope acceleration

Mobilized regolith layer motion variables
xl = horizontal slope position
yl = vertical slope position
vx1 = horizontal slope velocity
vy1 = vertical slope velocity
ax1 = horizontal slope acceleration
ay1 = vertical slope acceleration

Mobilized regolith layer property variables
coh = regolith layer cohesion value
tfrs = tangent of static angle of friction
tfrd = tangent of dynamic angle of friction
mass = mass per unit area of regolith layer
ns = normal stress per unit area
ts = shear stress per unit area (static case)
td = shear stress per unit area (dynamic case)

Binary flags (either 0 or 1)
cnty = vertical contact flag (between layer and slope)
cntx = horizontal contact flag (between layer and slope)
dam = "damage" flag for cohesion between regolith layer and slope

Other variables:
dtv = differential velocity between slope and layer
std = a sign function (extracts either -1 or +1)
small = a very small number, used to prevent division by zero errors

The below program section is part of a "loop," which moves in small delta-t (dt) time steps through the provided synthetic seismogram. The steps below are done at each time step. The basic goal is to find the new accelerations on the regolith layer (both vertical and horizontal) and then double integrate to find the new layer velocity and position at each time step.
Note: if the layer and the slope are in contact with each other (either vertical or horizontal, i.e. no ballistic motion or sliding), then the layer is constrained to move along with the slope, until such time as this contact is broken.

cc ----- find new layer parameters ----- 

cc find vertical accelerations 
cc no contact (layer free fall) 
cc if (cnty.eq.0) then 
cc ayl = gv 
cc endif 
cc contact and no launch (the two move together) 
cc if (cnty.eq.1) then 
cc if (av.ge.gv) then 
cc ayl = av 
cc endif 
cc contact and launch (slope drops out from under the layer) 
cc if (av.lt.gv) then 
cc ayl = gv 
cc cntx = 0 
cc cnty = 0 
cc dam = 0.0d0 
cc endif 
cc endif 

cc find normal stress on the layer 
cc if (cnty.eq.1) then 
cc ns = av-gv 

if (ns.lt.0.0d0) ns = 0.0d0
endif

c if layer is launched, no normal stress
if (cnty.eq.0) then
    ns = 0.0d0
endif

c find shearing resistance (static and dynamic)
ts = ((dam*coh)+(mass*ns*tfrs)) / mass
td = ns*tfrd

c find horizontal accelerations
c no contact
if (cnty.eq.0) then
    axl = gh
endif

c contact and sliding
if (cnty.eq.1).and.(cntx.eq.0) then
    dtv = vh-vxl
    std = (dtv+small)/(abs(dtv)+small)
    axl = gh + td*std
endif

c contact and no sliding
if (cnty.eq.1).and.(cntx.eq.1) then
    if (abs(gh-ah).le.ts) then
        axl = ah
    endif

c contact and begin sliding
if (abs(gh-ah).gt.ts) then
    dtv = ah
    std = (dtv+small)/(abs(dtv)+small)
    axl = gh + td*std
    cntx = 0
    dam = 0.0d0
endif
endif

c ----- perform motion integration ----- 

c record old position markers
xlp = xl
ylp = yl
vxlp = vxl
vylp = vyl

c update the velocities and position
if (cntx.eq.0) then
    vxl = vxlp + axl*dt
    xl = xlp + vxl*dt
endif
if (cnty.eq.0) then
    vyl = vylp + ayl*dt
    yl = ylp + vyl*dt
endif

c ----- check for landings and sliding stops -----
c perform landing detection & reset vertical contact flag
  if ((cnty.eq.0).and.(yl.le.pv)) then
      cnty = 1
  endif

c perform slide stop detection and reset sliding flag
c determine if block is in contact with slope
  if ((cnty.eq.1).and.(ctx.eq.0)) then

c determine if shearing stress is less than dynamic stress
  if (abs(gh-ah).le.td) then

  endif
  if ((vxl.p.gt.vh).and.(vxl.le.vh)) then
      ctx = 1
  endif
  if ((vxl.p.lt.vh).and.(vxl.ge.vh)) then
      ctx = 1
  endif
  endif
endif

--- resets for non-sliding layer ---

c no initial motion resets for layer
c layer continues to move with slope
  if (dam .eq. 1.0d0) then
      xl = ph
      yl = pv
  endif
vxl = vh
vyl = vv
axl = ah
ayl = av
endif

c vertical contact reset for layer
c layer moves vertically with slope
if (cnty.eq.1) then
  yl = pv
  vyl = vv
  ayl = av
endif

c horizontal contact reset
c layer moves horizontally with slope
if (cntx.eq.1) then
  vxl = vh
  axl = ah
endif

Note: if the double integration were exact, these last steps would not be necessary, but since the numerical integration is not exact, a small difference in position and velocity would creep in, even though the layer and slope were supposed to be moving together. Therefore, at the end of each time step, these checks are done to ensure that if the layer and slope are supposed to be moving together at that time step, then their accelerations, positions, and velocities are appropriately identical.
REFERENCES


