Improving the Modeling of Impact Ejecta Behavior: The Effects of Gravity and Strength Near the Crater Rim. J. E. Richardson, Center for Radiophysics and Space Research, 310 Space Sciences Building, Cornell University, Ithaca, NY 14853, richardson@astro.cornell.edu.

Introduction: On July 4, 2005, the Deep Impact mission successfully collided a 370 kg impactor with the surface of Comet 9P/Tempel 1 at a speed of 10.2 km/sec. This impact produced a prominent solid ejecta plume: an inverted-conical particle cloud which remained attached to the comet's surface and slowly expanded over the course of 1.25 hours of observation. In our previous work [1], we described a method for modeling the behavior of impact ejecta launched during crater excavation from the surface of a small, irregular target body, specifically designed for this mission. However, this model has one particular weakness, as pointed out in [2]: that the effects of target gravity and strength on ejection velocities are only crudely approximated near the crater rim. This weakness has led to disparate interpretations of the DI event, particularly regarding the strength of the comet's surface. In this work we present recent advances which greatly improve our modeling of ejecta launch - especially near the crater rim - for both gravity and strength dominated impact events.

Idealized Ejecta Velocity Functions: Our ejecta model is based upon the pi-group scaling relations for impact cratering, derived through experiment and dimensional analysis over several years [3]. Most important are the scaling relations for particle ejection velocity \( v \) as a function of distance \( r \) from an impact site, in the gravity (1) and strength (2) regimes [4]:

\[
\begin{align*}
C_{vg} &= \sqrt{\frac{2}{g}} \left( \frac{\mu}{\mu + 1} \right) (3), \quad C_{vs} = \sqrt{\frac{2}{Ts}} \left( \frac{\mu}{\mu + 1} \right) (4),
\end{align*}
\]

where \( C_{Tg} \) and \( C_{Ts} \) come from the crater formation time relations:

\[
\begin{align*}
T_g &= C_{Tg} \sqrt{\frac{R_g}{g}} (5), \quad T_s &= C_{Ts} R_s \sqrt{\frac{\rho}{Y}} (6),
\end{align*}
\]

Due to the self-similarity of all gravity-dominated craters, the proportionality constant \( C_{Tg} \) is invariant across both similar and non-similar impacts, and has an experimentally determined value of 0.8-0.9 [2, 7]. Unfortunately, strength-dominated craters do not share this property and the value of \( C_{Ts} \) is only invariant across similar impacts. However, because Eqs. 1 and 2 are equivalent far from the crater rim, where both gravity and strength effects are small compared to the flow inertia, we can solve for the value of \( C_{Ts} \) for each impact environment, given a value of \( C_{Tg} \). This gives us two "idealized" ejecta velocity functions for a given impact (thin solid lines in Fig. 1). These functions, however, contain no mechanism for stopping crater growth, and will run out to infinitely large crater radii \( r \) to produce infinitely small ejection velocities \( v \).

Velocity Braking Function: To bring in the effects of gravity \( g \) and strength \( Y \) and halt the crater's growth, we take advantage of a concept described in the Maxwell Z-model of excavation flow [6]: namely, that the ejecta flow emerging from the surface at some radius \( r \) from the impact site represents a hydrodynamic stream-line (stream-tube in axial symmetry), which is steady state and incompressible. If we also assume that frictional forces are small compared to the forces of inertia, gravity, and strength (inviscid flow), we can use Bernoulli's principle at the point of ejecta emergence to form an energy balance equation:

\[
\frac{1}{2} \rho v_e^2 = \frac{1}{2} \rho v^2 - K_g \rho gr - K_s Y (7),
\]

where \( v \) is our idealized velocity, and \( v_e \) is the effective velocity that we desire. Beginning on the left, the first two terms describe the kinetic energy (or stagnation pressure) of the ejecta flow in a single stream-tube, assuming that upon emergence, the hydrostatic pressure in the flow is zero. The third term describes the mean amount of gravitation potential energy needed to loft each unit volume in the flow (a function of radius \( r \)), and the fourth term describes the amount of energy needed to fracture or "break loose" each unit volume in the flow (a function of target strength \( Y \)).
Using Eq. 1 for $v$ in Eq. 7, we solve for the value of $K_g$ by setting the target strength $Y$ to zero and letting the crater radius $r$ go to $R_g$ as the effective velocity $v_e$ goes to zero. This gives $K_g = C_{vg}v_e^2/2$. In similar fashion, we use Eq. 2 for $v$ in Eq. 7, and solve for the value of $K_s$ by setting the target gravity $g$ to zero and letting $r$ go to $R_s$ as $v_e$ goes to zero. This gives $K_s = C_{vs}v_e^2/2$. Thus, Eq. 7 in final form is (thick lines in Fig. 1):

$$v_e^2 = v^2 - C_{vg}gr - C_{vs}Y \frac{Y}{\rho}$$

(8).

The value of $K_g r$ represents a mean stream-tube excavation depth, and ranges from $r/9.0$ ($\mu = 0.40$) to $r/5.8$ ($\mu = 0.55$). This implies a maximum excavation depth of $r/4.5$ to $r/2.9$, which is reasonable (usual values range from $r/5$ to $r/4$).

The value of $K_s Y$ represents a post-shock "yield" strength for the target material, and due to the variant nature of $C_{Ts}$, can range from $Y/3$ to $Y/15$, with values of $Y/5$-$Y/10$ being typical. These values are less than the Tresca maximum shear stress criteria of $Y/2$, assuming the primary stress during excavation is shear stress, but might be reasonable for such post-shock, "pre-damaged" target material (further investigation is needed regarding this constant).

**Overturn Flap Behavior:** Another way to check our velocity braking function is to look for "overturn-flap" behavior near the rim of gravity-dominated events. This can be described functionally as the velocity needed to produce a hinge-like overturn of material about the transient crater rim (dotted line in Fig. 1). Our velocity braking function shows good agreement with this overturn-flap function over the final ~10% of crater growth in gravity-dominated events.

**Application:** With this new method in hand, our next step is to make direct comparisons between this model and experimental results, particularly near the crater rim. A re-analysis of the Deep Impact event using this improved model is also underway, to better constrain the strength of the comet's surface and better understand the evolution of the impact ejecta plume.

**References:**

**Figure 1:** Modeled ejecta velocities as a function of radial distance from the laboratory-like impact of a 1 mm glass bead striking a dry soil target at 5 km/sec in Earth $g$, shown in both linear (left) and log (right) forms. Legend: *Thin solid line* = idealized velocity curve; *Thin dotted line* = overturn-flap function, hinged at the calculated gravity-dominated crater radius; *Thick solid line* = zero-strength model velocity curve (gravity-dominated); *Thick dot-dash lines* = model velocity curves at strengths of (0 dot) 0.5 kPa, (1 dot) 5 kPa, (2 dot) 50 kPa, and (3 dot) 500 kPa; *Thin dashed lines* = calculated strength-dominated crater radii for these strength values.